THE TEETH OF

GEAR WHEELS.

INTRODUCTION.

Few mechanical subjects have attracted the attention of scientific men to such an extent, or are so intimately connected with mathematics, as the proper construction of the teeth of gear wheels, and, as a consequence, few can show such an advance as has here been made, from the rough cog wheel of not many years ago, to the neat cut gear of the present day.

It is not apparent wherein much further improvement is needed in our knowledge of the theory of the subject, but it is evident that much remains to be done towards its practical application, and to induce the working mechanic to understand and use the improvements that have been developed by the mathematician and the inventor. The theory seems to be full and well nigh perfect, but the mill-wright and the machinist still clings to imperfect rules and clumsy devices that should have been forgotten years ago, and few workmen have a clear knowledge of even the rudiments of the science which it is their business to apply to practical purposes.

It is the mathematical and scientific character of the subject that makes it so difficult to the practical man, who can understand but little of it as it is commonly presented in elaborate treatises or encyclopædias, and who takes but little interest in the study of a matter that bristles with strange characters and technical terms.

I have here undertaken to address the workman as well as the man of science, and have felt obliged to leave out nearly everything that cannot be treated in a plain, descriptive manner, to use language that any intelligent man can understand, and to refer to more pretentious works than this for demonstrations, or unessential details.

A volume of a thousand pages would not properly present the whole subject, and this little pamphlet can deal only with the main principles and prominent points. It is not a treatise, it is a hand-book that does not pretend to cover the whole ground, and its principal object is to present the new odontographs, which I believe to be superior to those heretofore in use for the purpose of designing the teeth of gear wheels.

FIRST PRINCIPLES.

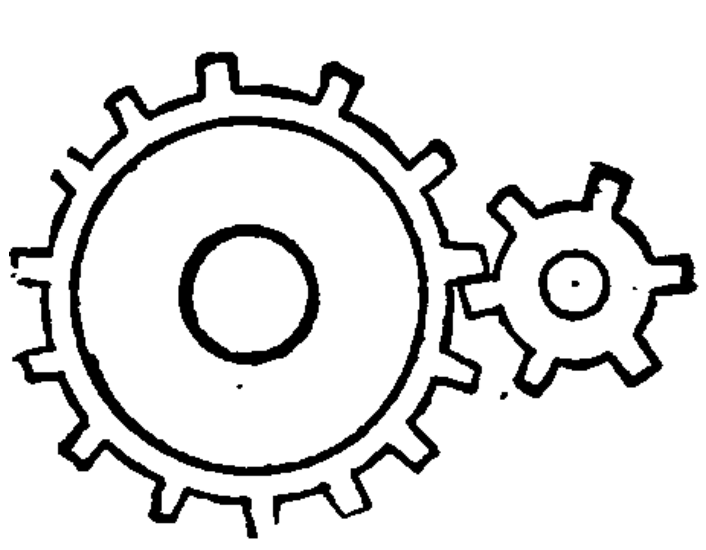


FIG. 1.
THE ORIGINAL GEAR WHEEL.

The original gear wheel had pins or projections for teeth, of any form that would serve the general purpose and communicate an unsteady motion from one wheel to another.

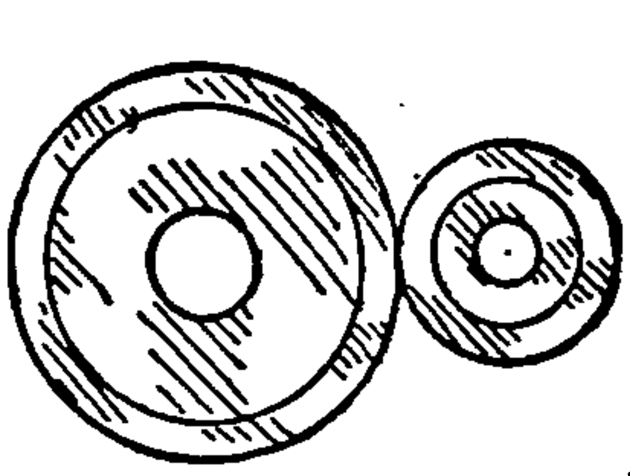


FIG. 2. FRICTION WHEELS.

The perfect gear wheel is the friction wheel, communicating a smooth, uniform, rolling motion, by means of the frictional contact of its surface. It is, in fact, a gear wheel with a great many very small, weak, and irregular teeth.

The whole aim and object of the science of the teeth of gear wheels is to increase the size and strength of these teeth without destroying the uniformity of the motion they transmit, and this is accomplished by studying the shape of the teeth, and giving their bearing surfaces the curved outline that is found to produce the desired result.

There are an infinite number of curves that will meet the requirement, but only two, the epicycloid and the involute, are of any practical importance, or in actual use.

THE EPICYCLOIDAL TOOTH.

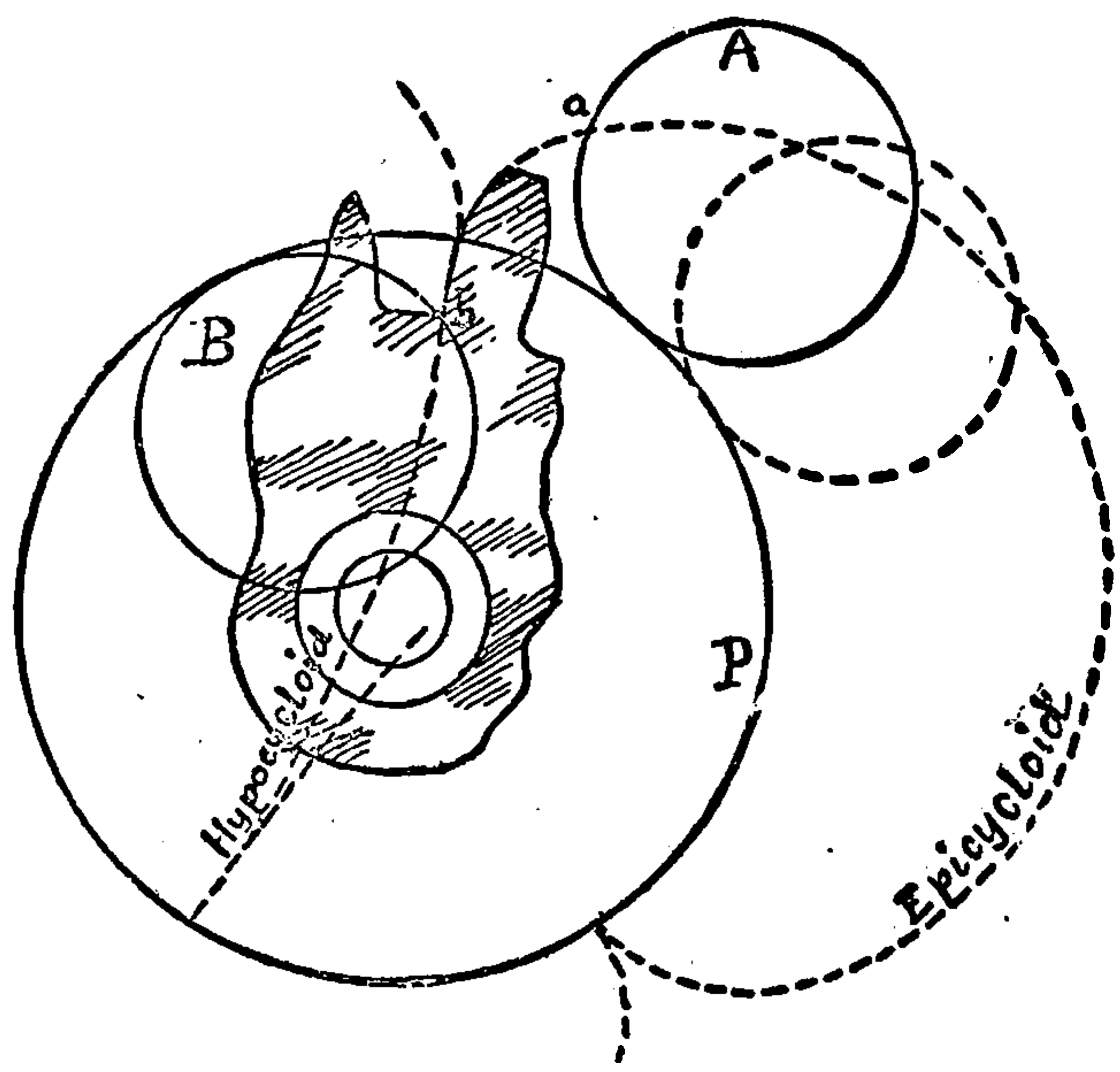


FIG 3. THE EPICYCLOIDAL TOOTH.

The epicycloidal or double curve tooth has its bearing surface formed of two curves, meeting at the pitch line P, which corresponds to the working circle of the perfect gear wheel of fig. 2.

If a small circle, a, be rolled around on the outside of the pitch circle, p, a fixed tracing point, a, in its edge, will trace out the dotted line called an epicycloid, and a small part of this curve near the pitch line, usually one sixth of its full height, forms the face of the tooth.

Similarly, if a small circle, B, be rolled around on the inside of the pitch line, its tracing point, b, will describe the internal epicycloid, or hypocycloid, a small portion of which is used for the flank of the tooth.

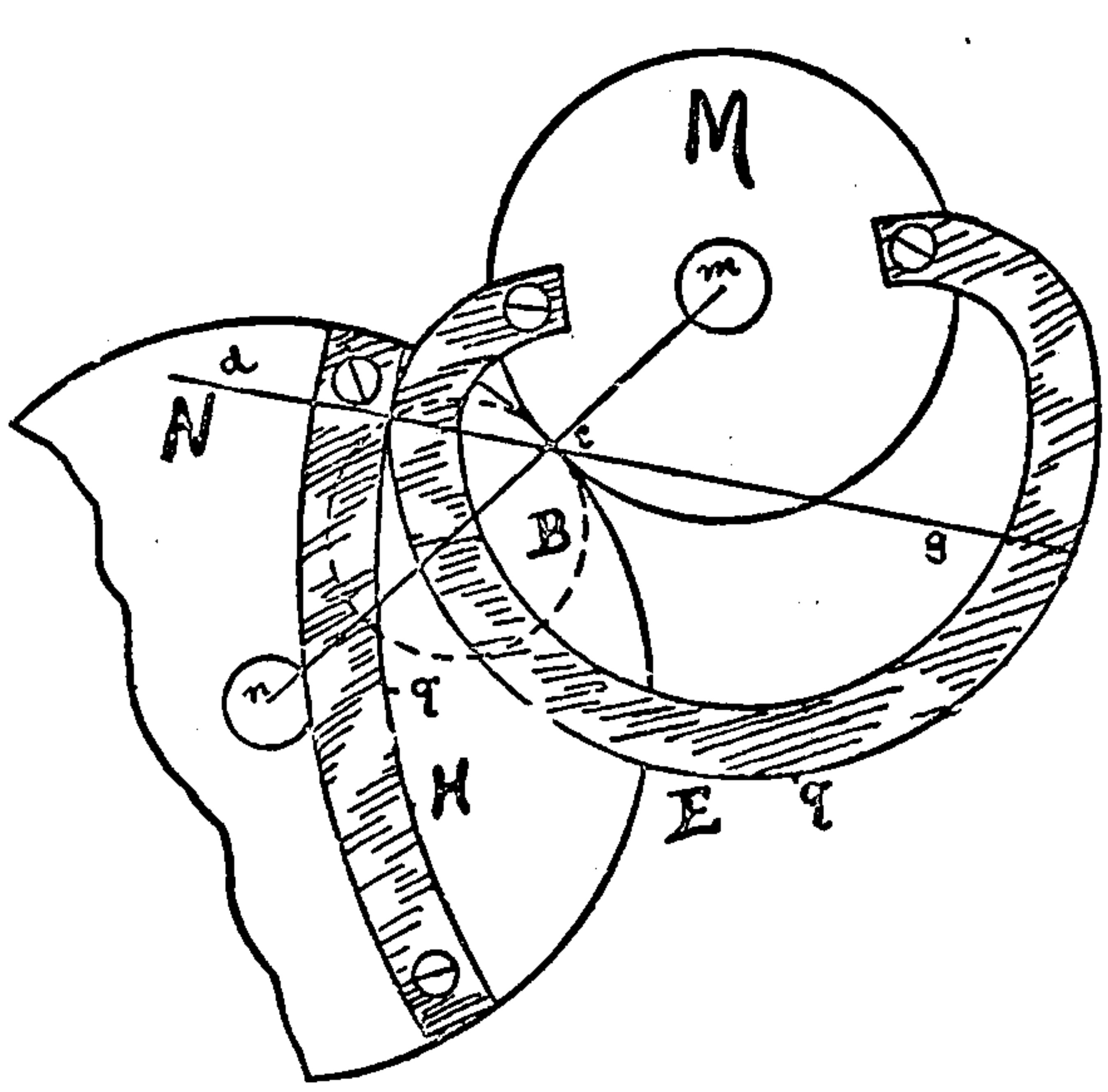


FIG .4 WHOLE EPICYCLOIDAL TEETH.

If a projection be formed on the friction wheel fig. 4, the curved outline of which is a whole epicycloid E, and a depression be formed in the wheel N having a whole hypocycloid H for its outline, then, if both curves have been formed by the same describing circle B, it can be mathematically demonstrated that the two curves will just touch and slide on each other, without separating or intersecting, while the two friction wheels roll together.

The reverse of this fact is also true; that, if one wheel drives another by means of an epicycloidal projection on it working against a hypocycloidal depression in the other, both curves being formed by the same describing circle, the two wheels will roll together as uniformly as if driven by frictional contact, and it is this peculiar property of the epicycloid that gives it its value for the purpose in hand.

The pressure acting between the two curves is in the direction of the line dg, is direct only at the start, and becomes more and more oblique, until, when the middle points, q q, come together, and beyond, there is no driving action at all. This defect forbids the use of the whole curve and we can use but a small portion of it near the pitch line. Another projection and depression must be formed so near the first that they will come into working position before the first pair are out of contact, thus forming the theoretically perfect but incomplete gears of fig. 5.

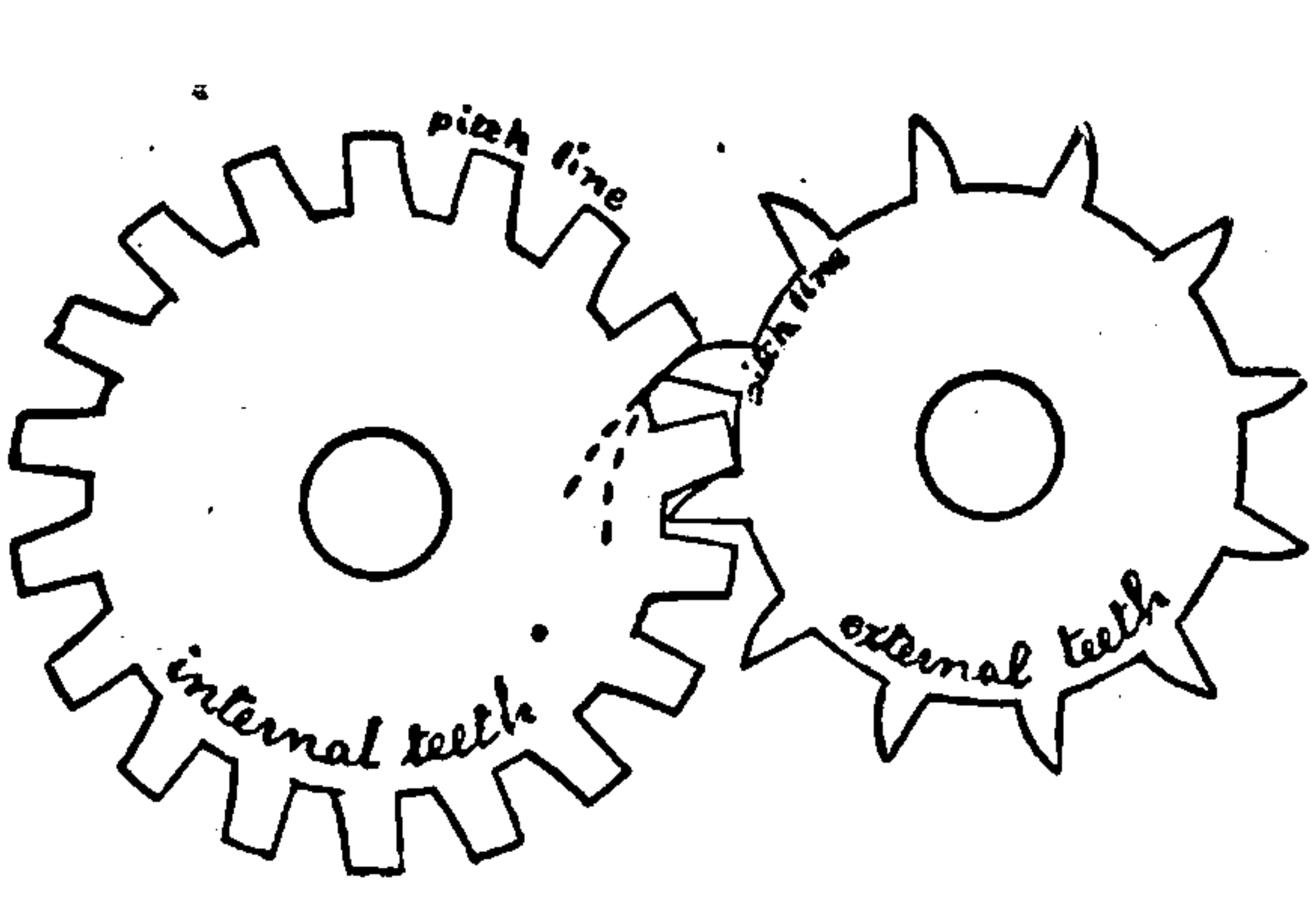


FIG. 5. INCOMPLETE EPICYCLOIDAL TEETH.

Practical requirements still further modify the apparent shape of the tooth, for it is desirable that the wheels shall work in either direction, and that they shall be interchangeable, so that any one of a set of several shall work with any other of that set.

This can be accomplished only by making the curves face both ways, and by putting both projections and depressions on each gear, thus forming the familiar tooth of fig. 3.

THE INTERCHANGEABLE SET.

If all the curves of a set of several gears, both the faces and the flanks of each gear, are described by the same rolling circle, the set will be interchangeable, and any one will work perfectly with any other.

This is a property of the greatest practical importance, and interchangeable sets should come into as universal use on heavy mill work as with cut gearing. It is the only system that will allow the use of a set of ready made cutters, and is therefore essential to the economical manufacture of cut gear wheels.

The diameter of the rolling circle is usually made half the diameter of the smallest gear of the set, and that gear will have straight radial lines for flanks.

The set in almost universal use and adopted for all the odontographs, has twelve teeth in its smallest gear, but there is a tendency to change this well established system, and create confusion for which the writer can see no adequate excuse, by the adoption of a pinion of fifteen teeth as the base or smallest gear. It may be admitted that as large a base as possible should be used, but the change from twelve to fifteen seems to be unwarranted in view of the confusion it creates by the abrupt change from an old and good rule to a new one that is a mere shade better, and the trouble it makes with small pinions of eight to twelve teeth.

RADIAL FLANK TEETH.

If the internal curves, or flanks, of a pair of gears that are to run together are on each radial straight lines described by a rolling circle of half its pitch diameter, and the rolling circle that describes the flanks of one gear is used to describe the faces of the other gear, then, the two gears will form a pair fitted to each other and not interchangeable with other gears.

This style of gear is very often used under the erroneous impression that it is the best possible form, and will give the least possible friction and thrust on the bearings, but the saving in friction over the interchangeable form would be an exceedingly difficult thing to measure by any practicable method, although it can be mathematically demonstrated to be a fact, and the slender roots of such teeth make them weaker and much inferior to the others. The odontograph figures show both a pair of these gears, and the same pair on the interchangeable plan, also, by the dotted lines on the former figure, the shapes as they would be on the interchangeable plan. It is plainly seen that the interchangeable faces are but a shade more rounding, while their flanks are so curved that the teeth are much stronger at the roots. The larger the describing circle, the less the theoretical thrust and friction, and if the flanks were formed by a describing circle of more than half the diameter of the gear, the teeth would be undercurved, the friction less, and their strength less, than that of the radial flank tooth.

In practical matters it is a good plan to give first place to practical points, and not to take too much notice of minute theoretical advantages, and there is no good reason, that will bear the test of experiment, for adopting the radial flank, non-interchangeable, and weak tooth, in preference to the strong tooth of the interchangeable system.

THE PITCH.

The pitch is a term used to designate the size of the tooth, and is either circular or diametral.

THE CIRCULAR PITCH or more properly the circumferential pitch, is the actual distance from tooth to tooth measured along the curve of the pitch line, and is expressed in inches, as $\frac{5}{8}$ inch pitch, $1\frac{1}{2}$ inch pitch, etc.

The table gives the proper pitch diameter of a gear of any given number of teeth, and one inch circular pitch. The tabular numbers must be multiplied by any other pitch that is in use.

Formerly, the circular pitch was the only one known, but it has deservedly gone out of use on cut gears, and it is hoped may soon be abandoned altogether. It is a clumsy, awkward, and troublesome device on either large or small work, having its origin in the ignorance of the past, and owing its existence not to any perceptible merit, but to habit, and the natural persistence of an established custom.

With the circular pitch the relation between the pitch diameter of the gear, and the number of teeth on it, is fractional. If the diameter is a convenient quantity, such as a whole number of inches, the pitch must be an inconvenient fraction, and if the pitch is a handy part of an inch, the diameter will contain an unhandy decimal.

With the circular pitch there is no one length of tooth that is better than any other, and consequently there is no agreement upon that point. Each maker is at liberty to chose his own distance at random, and whatever he choses is as good as any other.

Its worst feature is that it leads to endless errors, for the average mechanic appreciates convenience more than accuracy, and will stretch his figures to suit his facts, with a botch as the common result.

A millwright figures out a diameter of 22.29 inches for a gear of one inch pitch and 70 teeth, and failing to make such a clumsy figure fit his work or his foot rule, and thinking a quarter of an inch or so to be of no importance, he lets it go at 22 whole inches. The same process on its mate of 15 teeth gives a 5 inch gear instead of one of 4.78 inches diameter, and the pair will never run or wear together properly. His only alternatives are to adopt the clumsy true diameters, or else use the clumsy figure .988 inch for his pitch.

Again, he is apt to apply a carpenter's rule directly to the teeth of the gear he is to repair or match, and naturally takes the nearest convenient fraction of an inch as his measurement, when the real pitch may be just enough different to spoil the job.

There is no reason whatever for using the circular pitch, unless the work to be done is to match work already in use.

THE DIAMETRAL PITCH is an immense improvement on the old fashioned circular pitch. It is not a measurement, but a number, or ratio. It is the number of teeth on the gear, for each inch of its pitch diameter, and its merit is that it establishes a convenient and manageable relation between these two principal elements, so that the calculations are of the simplest description and the results convenient and accurate.

The product of the pitch and the pitch diameter is equal to the number of teeth, and the number of teeth divided by the pitch is equal to the pitch diameter. A gear of 15 inches diameter and 2 pitch has 30 teeth, and a gear of 27 teeth of 4 pitch has a pitch diameter of $6\frac{3}{4}$ inches.

The rule that the length of the tooth is two pitch parts of an inch, $\frac{2}{4}$ or $\frac{1}{2}$ an inch for 4 pitch, $\frac{2}{2}$ or 1 inch for 2 pitch, etc. is so simple and so much better than any other that it is never disputed, and is in universal use.

The circular and diametral pitches are connected by the relation

$$c \times p = 3.1416.$$

or, the product of the circular and the diametral pitch is the number 3.1416.

THE ADDENDUM.

For reasons expressed above we can use but a small part of the epicycloidal curve near the pitch line, limiting it by a circle drawn at a distance inside or outside of the pitch line called the addendum. The outside limit need not be the same as the inside limit, but it is customary to make them equal.

When the diametral pitch is used, the length of the addendum is always one pitch part of an inch, as $\frac{1}{4}$th inch for 4 pitch, $\frac{1}{3}$rd inch for 3 pitch, etc. If we use the same proportion for circular pitches the addendum will be $\frac{1}{3.1416}$ circular pitch, and the value $\frac{1}{3}$rd of the circular pitch may be adopted as the most convenient for use.

THE CLEARANCE.

Theoretically, the depression formed inside the pitch line should be only as deep as the projection outside of it is high, but to allow for practical defects in the making or in the adjustment of the teeth, and to provide a place for dirt to lodge, the depression is always deeper than theory requires by an amount called the clearance. The amount of the clearance is arbitrary, but the sixteenth part of the depth of the tooth is a convenient and customary measure, or $\frac{1}{24}$th of the circular pitch, and 1 divided by 8 times the diametral pitch. The following tables will be convenient and save calculation:

CLEARANCE FOR CIRCULAR PITCHES.

Circular pitch.	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	3
Clearance.	.02	.03	.03	.04	.04	.05	.05	.06	.06	.07	.08	.09	.10	.12

CLEARANCE FOR DIAMETRAL PITCHES.

Diametral pitch.	6	5	4	$3\frac{1}{2}$	$3\frac{1}{4}$	3	$2\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{1}{4}$	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1
Clearance.	.02	.03	.03	.04	.04	.04	.05	.05	.06	.06	.08	.09	.10	.12

THE BACKLASH.

When wooden cogs or rough cast teeth are used, the inevitable irregularities require that the teeth should not pretend to fit closely, but that the spaces should be larger than the teeth by an amount called the backlash. The amount of the backlash is arbitrary, but it is customary to make it about equal to the clearance.

Cut gears should have no allowance for backlash, and involute teeth need less backlash than epicycloidal teeth.

PITCH DIAMETERS.

FOR ONE INCH CIRCULAR PITCH.

FOR ANY OTHER PITCH, MULTIPLY BY THAT PITCH.

T.	P.D.	T.	P.D.	T.	P.D.	T.	P.D.
10	3.18	33	10.50	56	17.83	79	25.15
11	3.50	34	10.82	57	18.15	80	25.47
12	3 82	35	11.14	58	18.47	81	25.79
13	4.14	36	11.46	59	18.78	82	26.10
14	4 46	37	11.78	60	19.10	83	26.43
15	4.78	38	12.10	61	19.42	84	26.74
16	5.09	39	12.42	62	19.74	85	27.06
17	5.40	40	12.74	63	20.06	86	27.38
18	5.73	41	13.05	64	20.38	87	27.70
19	6,05	42	13.37	65	20.69	88	28.02
20	6.37	43	13.69	66	21.02	89	28.34
21	6 69	44	14.00	67	21.33	90	28.65
22	7.00	45	14.33	68	21.65	91	28.97
23	7.32	46	14.65	69	21.97	92	29.29
24	7.64	47	14.96	70	22.29	93	29.60
25	7.96	48	15.28	71	22.60	94	29.93
26	8.28	49	15.60	72	22.92	95	30.25
27	8.60	50	15.92	73	23.24	96	30.56
28	8.90	51	16.24	74	23.56	97	30.88
29	9.23	52	16 56	75	23.88	98	31.20
30	9.55	53	16.87	76	24.20	99	31.52
31	9.87	54	17.19	77	24.52	100	31.84
32	10.19	55	17.52	78	24.83		

THE EPICYCLOID.

THEORETICAL FORMATION.

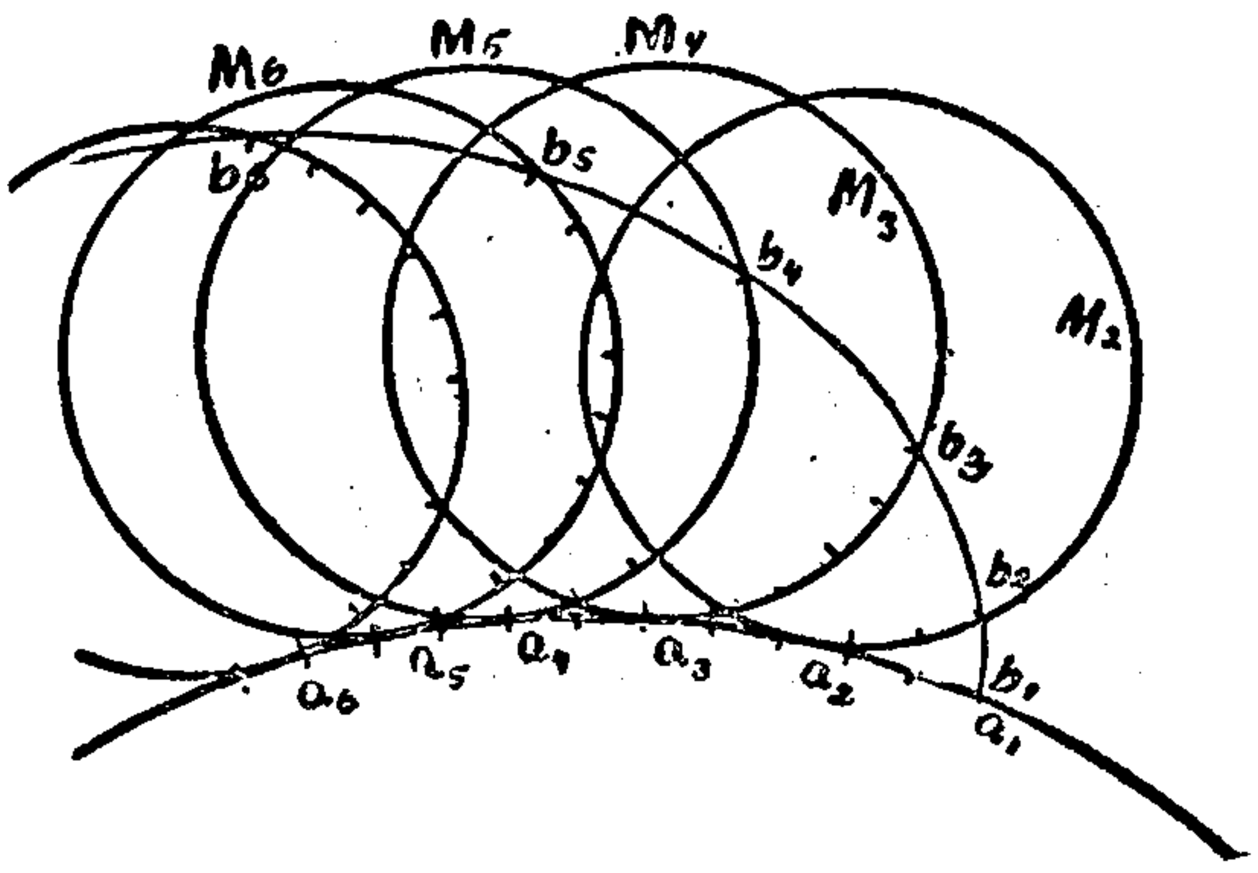


FIG. 6. THE EPICYCLOID.

The true epicycloid, shown by fig. 6, is perpendicular to the pitch line at the origin a, and forms an endless series of lobes about it, as in fig. 3.

The most convenient and simple process for drawing it, is to step it off with the dividers. Several describing circles, M^1 to M^5, are drawn at random; steps are made, as shown by the figure, from the origin a^1 to past each tangent point, a^1 to a^5, and then the same number back, around each circle, to locate the several points, b^2 to b^5, on the curve, which is then drawn by hand through the points, and is accurately in place if the steps are small.

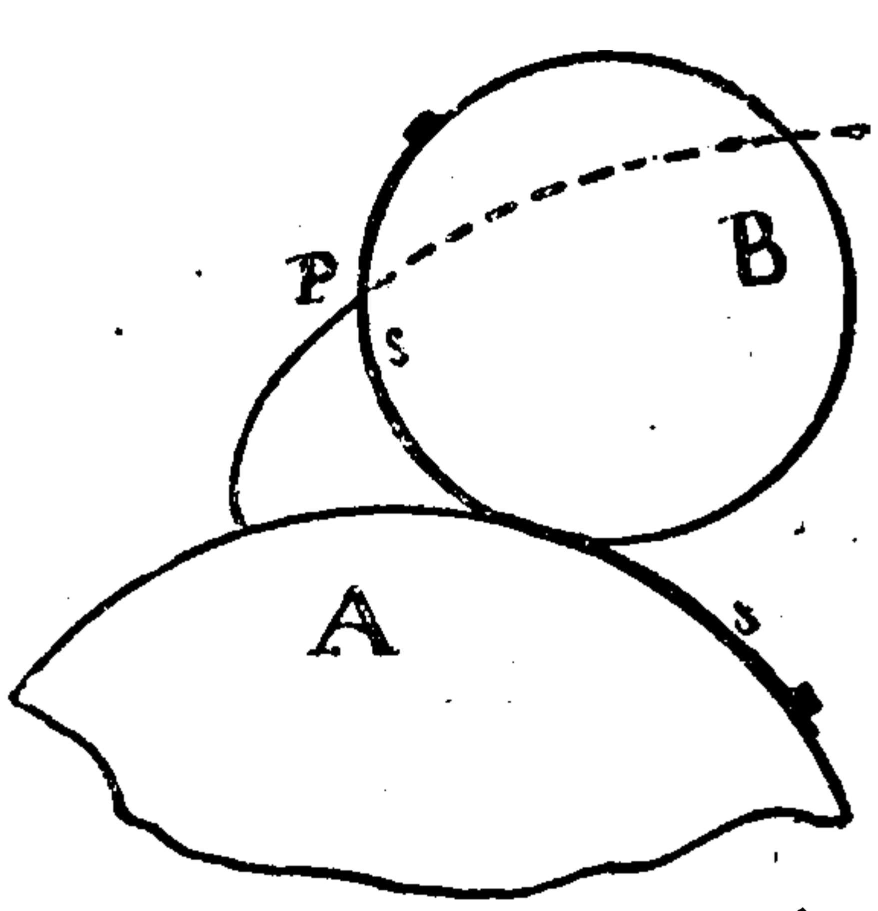


FIG. 7. THE EPICYCLOIDAL ENGINE.

By the mechanical method for drawing the curve, the describing circle, B, is rolled around the pitch circle A, and a tracing point or pencil P, draws the curve. A steel ribbon s, is fastened to the templets at each end, and assists in keeping them in place.

This process is the main principle of the epicycloidal engine, which carries a scribing tool, or a rotary cutter at p, to trace or cut out a templet that is then used in forming gear teeth or gear cutters.

It is, of course, the most accurate method known, but it is not available for ordinary purposes, for unless the templets are well made and skillfully handled, the resulting curve will be poorly drawn, and the method, although simple in principle, may be considered difficult in its practical application.

PRACTICAL FORMATION.

Of course nothing but the perfect curve will answer its purpose with perfect accuracy, but the epicycloid is a peculiar curve which cannot be accurately drawn by any simple process, or with common instruments, particularly when the teeth are small, and it is customary to use arcs of circles or other curves, which approximate as nearly as possible to the true curve.

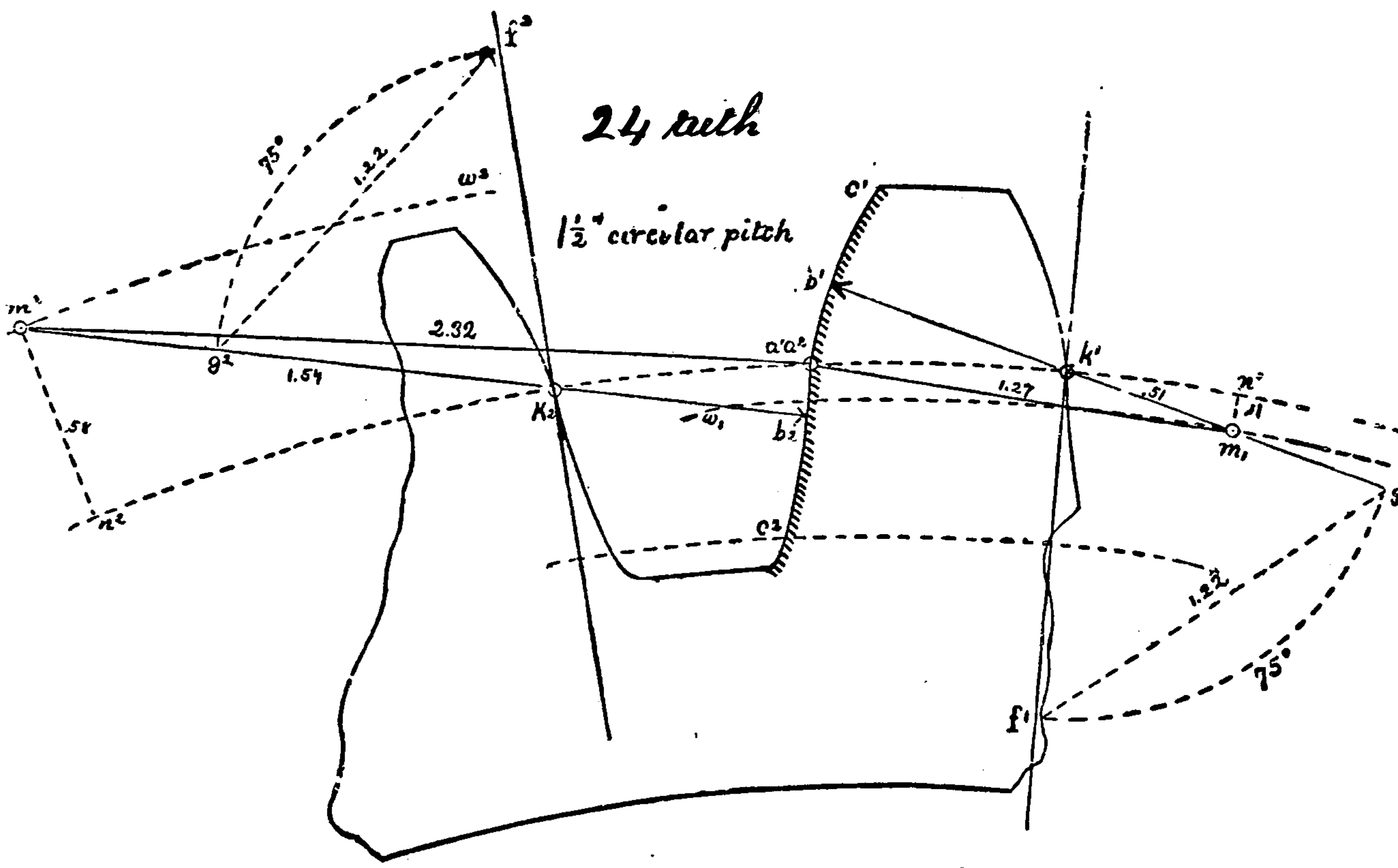
Such an arc can be made to agree with the curve so closely that it is a needless refinement to be more particular for most practical purposes, such as drafting teeth, making wooden cogs or patterns for cast teeth, or even the templets for shaping gear-cutters and planing bevel gear teeth.

Some makers of rough cast or heavy planed gearing go to g expense to construct the (supposed to be) theoretically true epicycloid, by means of rolling circles. This practice looks very much indeed like accuracy, but if he had an absolutely true curve as a templet, supposing he could make such a thing, the maker of this class of work could not produce from it a working tooth more nearly perfect than if the templet was properly constructed of circular arcs. It is labor lost to lay out teeth to the thousandth of an inch, that must be constructed with ordinary hand or machine tools, or shaped with a chisel and mallet.

Furthermore, it is a question if the delicate processes and epicycloidal engines used for the finest cut gear work, can serve practical purposes and construct templets to work from, better than intelligent and skillful hand-work. It is a fact that the best work in this line is made from templets that are laid out by theory, but dressed into shape and perfected by hand and eye processes.

ODONTOGRAPHS.

Many arbitrary or "rule of thumb" methods for shaping gear teeth have been proposed, but they are generally worthless, and reliance should be placed only on such as are founded on the mathematical principles of the curve to be imitated. Of these only three are known to the writer.



THE WILLIS ODONTOGRAPH is a method for finding the center m of the circle which is tangent to the epicycloid a b c, at the point b, where it is cut by a line b m, which passes through the adjacent pitch point k, and makes the angle g k f=75° with the radial line k f.

The radius used, is not the line m b, but the more convenient line m a. The instrument is nothing whatever but a piece of card or sheet metal cut to the angle of 75°, which is laid against the radial line k f, as a guide for drawing the line k m. The center distance k m, to be laid off along the line thus drawn is given by a table that accompanies the instrument.

No instrument is necessary, for the line k m may be placed by drawing the arc f g with a radius of one inch, and laying off the chord f g=1.22 inch. The tabular distance k m can be readily computed from

$$k_1\ m_1 = \frac{c}{20.3} \cdot \frac{t}{t+12}$$

$$k_2\ m_2 = \frac{c}{2.03} \cdot \frac{t}{t-12}$$

in which c is the circular pitch in inches, and t is the number of teeth in the gear.

The Willis odontograph, as found in use, is confined to the single case of an interchangeable series running from twelve teeth to a rack, but for any possible pair of gears the angle becomes

$$g\ k\ f = 90^0 - \frac{180^0}{s}$$

and
$$k_1\ m_1 = \frac{s\ c}{.628} \cdot \frac{t}{t+s} \cdot \sin. \frac{180^0}{s}$$

$$k_2\ m_2 = \frac{s\ c}{.628} \cdot \frac{t}{t-s} \cdot \sin. \frac{180^0}{s}$$

in which t is the number of teeth in the gear being drawn and s the number in the mate.

The accuracy of the Willis circular arc will be examined further on.

THE IMPROVED WILLIS ODONTOGRAPH.

EPICYCLOIDAL TEETH.

TWELVE TO RACK. INTERCHANGEABLE SERIES.

NUMBER OF TEETH IN THE GEAR.		FOR ONE DIAMETRAL PITCH. For any other pitch, divide by that pitch.				FOR ONE INCH CIRCULAR PITCH. For any other pitch, multiply by that pitch.			
		FACES.		FLANKS.		FACES.		FLANKS.	
Exact.	Intervals.	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.
12	12	2.30	.15	∞	∞	.73	.05	∞	∞
13½	13–14	2.35	.16	15.42	10.25	.75	.05	4.92	3.26
15½	15–16	2.40	.17	8.38	3.86	.77	.05	2.66	1.24
17½	17–18	2.45	.18	6.43	2.35	.78	.06	2.05	.75
20	19–21	2.50	.19	5.38	1.62	.80	.06	1.72	.52
23	22–24	2.55	.21	4.75	1.23	.81	.07	1.52	.39
27	25–29	2.61	.23	4.31	.98	.83	.07	1.36	.31
33	30–36	2.68	.25	3.97	.79	.85	.08	1.26	.26
42	37–48	2.75	.27	3.69	.66	.88	.09	1.8	.21
58	49–72	2.83	.30	3.49	.57	.90	.10	1.0	.18
97	73–144	2.93	.33	3.30	.49	.93	.11	1.5	.15
290	145–rack.	3.04	.37	3.18	.42	.97	.12	1.2	.13

THE IMPROVED WILLIS ODONTOGRAPH.

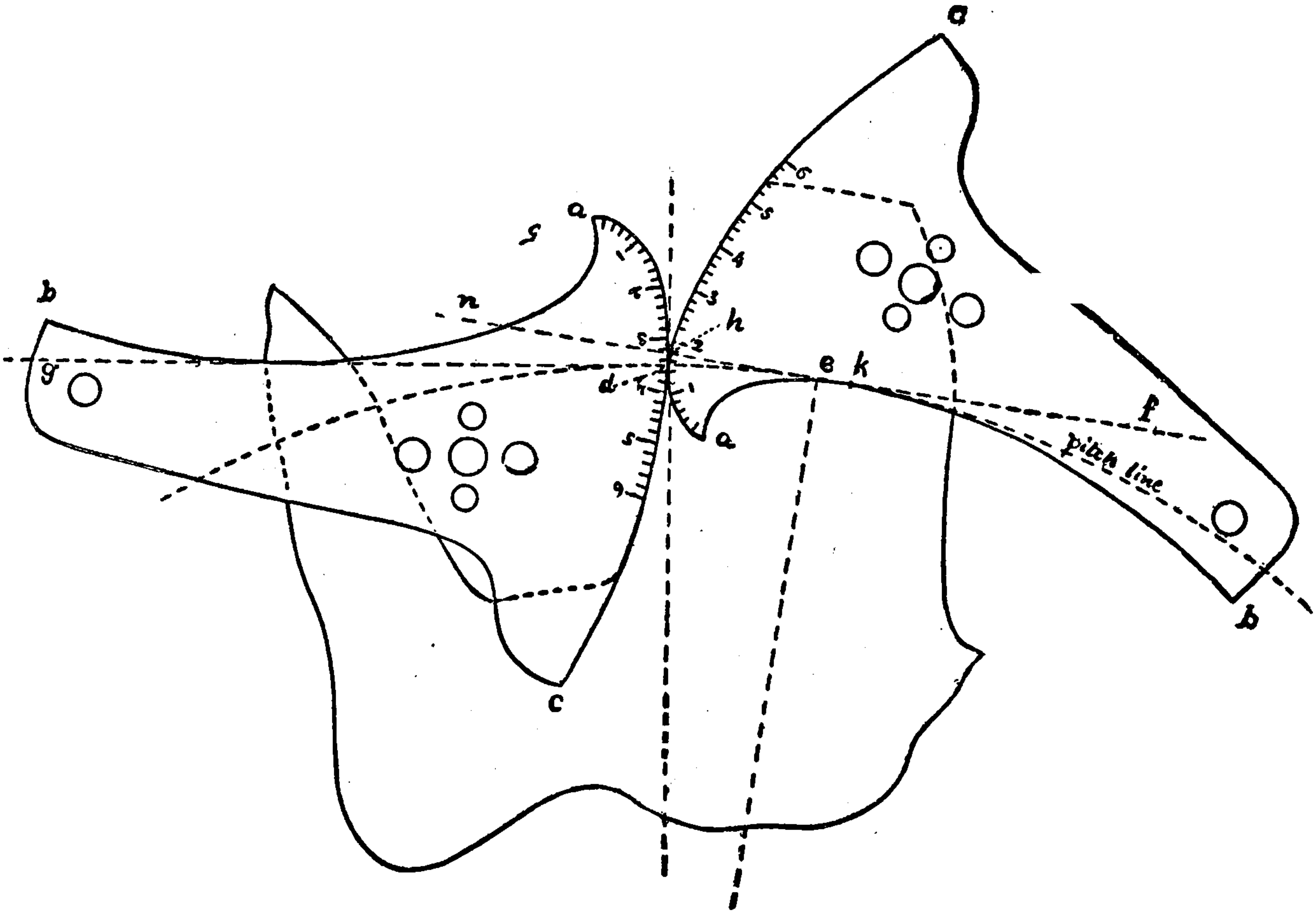
I have carefully calculated the distances m_1 n_1 and m_2 n_2 of the circles of centers from the pitch line, and also the radii a_1 m_1 and a_2 m_2, and have arranged them in the table above, so that the data resulting from the usual process can be obtained without the usual labor.

This improved Willis process will produce exactly the same circular arc as the usual method, with the same theoretical error, but its operation is simpler and less liable to errors of manipulation.

By the usual process it is necessary to draw two radial lines, and to lay off a line at an angle with each. The tabular distances laid off on these lines, will locate the two centers. The two circles of centers are then drawn through them, and the dividers set to the radii to be used.

By the new process the circles of centers are drawn at once without preliminary constructions, at the tabular distances from the pitch line, and the table also gives the radii to be taken on the dividers. No special instrument is required, no angles or special lines are drawn to locate the centers, and the chance of error is much less.

This process, however, is not as correct, and is no simpler or more convenient than the new odontographic process given further on.



ROBINSON'S TEMPLET ODONTOGRAPH.

This ingenious instrument, the invention of Prof. S. W. Robinson of the Ohio State University at Columbus, is based on the fact that some part of a certain curve of uniformly increasing curvature, called the logarithmic spiral, can be made to agree with the true curve of a gear tooth with a degree of approximation that is very precise.

It is a sheet metal templet having a graduated curved edge a c, shaped to a logarithmic spiral, and a hollow edge a b shaped to its evolute, an equal logarithmic spiral.

To apply the instrument, draw a radial line from the pitch point d on the pitch line, and another from e, the center of the tooth, and then draw tangents d g and n e f, square with the radial lines.

The instrument is then so placed that a certain graduation, given by accompanying tables, is at the point h on the tangent n e f, while the graduated edge a c, is at the pitch point d, and the hollow edge a b, just touches the tangent line n e f at k, and then the face of the tooth is drawn with a pen along the graduated edge. The flank is similarly located by placing the instrument so that a certain other graduation is at the pitch point d, while its hollow edge touches the tangent line g d.

The full theory of this instrument would be out of place here, but may be found in No. 24 of Van Nostrand's Science Series, or in Van Nostrand's Magazine for July, 1876.

A NEW ODONTOGRAPH.

Having frequently to apply the Willis Odontograph, it occurred to me that the process would be much simplified and much time and labor saved if the location of the circles of centers and the lengths of the radii were computed and tabulated, thus forming the improved Willis method already described.

It was then evident that the process would be precisely the same, and the result much improved, if the centers tabulated were the centers of the nearest possible approximating circles, rather than of the Willis circles, and I have embodied this idea in the following tables.

I have carefully computed, by accurate trigonometrical methods, and have tabulated the location of the center of the circular arc that passes through the three most important points on the curve, at the pitch line a, fig. 9, at the addendum line k, and the point e, half way between.

The tables locate this center directly, giving its distance from the pitch line, and from the pitch point.

The circles of centers are drawn at the tabular distances "dis" inside and outside the pitch lines, and all the faces and flanks are drawn from centers on these circles, with the dividers set to the tabular radii "rad."

The tables are arranged in an equidistant series of twelve intervals. For ordinary purposes the tabular value for any interval can be used for any tooth in that interval, but for greater precision it is exact only for the given "exact" number, and intermediate values must be taken for intermediate teeth.

The tables are arranged for both the diametral and circular pitch systems. The former is much the more manageable and should be used when the work is not to interchange with work already made on the latter system.

The first table, giving an interchangeable set, from twelve teeth upwards, is the one for general use.

The second, or radial flank table, is inserted because teeth are sometimes drawn that way, but, as before explained, they are weak, not interchangeable, and but a mere shade more direct in their action than the interchangeable style.

ACCURACY OF THE ODONTOGRAPH.

The assertion is often made that no circular arc can be made to do duty for the epicycloid, except for rough work, but it can be shown that the statement is not true if applied to the new method, for few mechanical processes can be made to work closer to a given example, than this arc is close to the true curve.

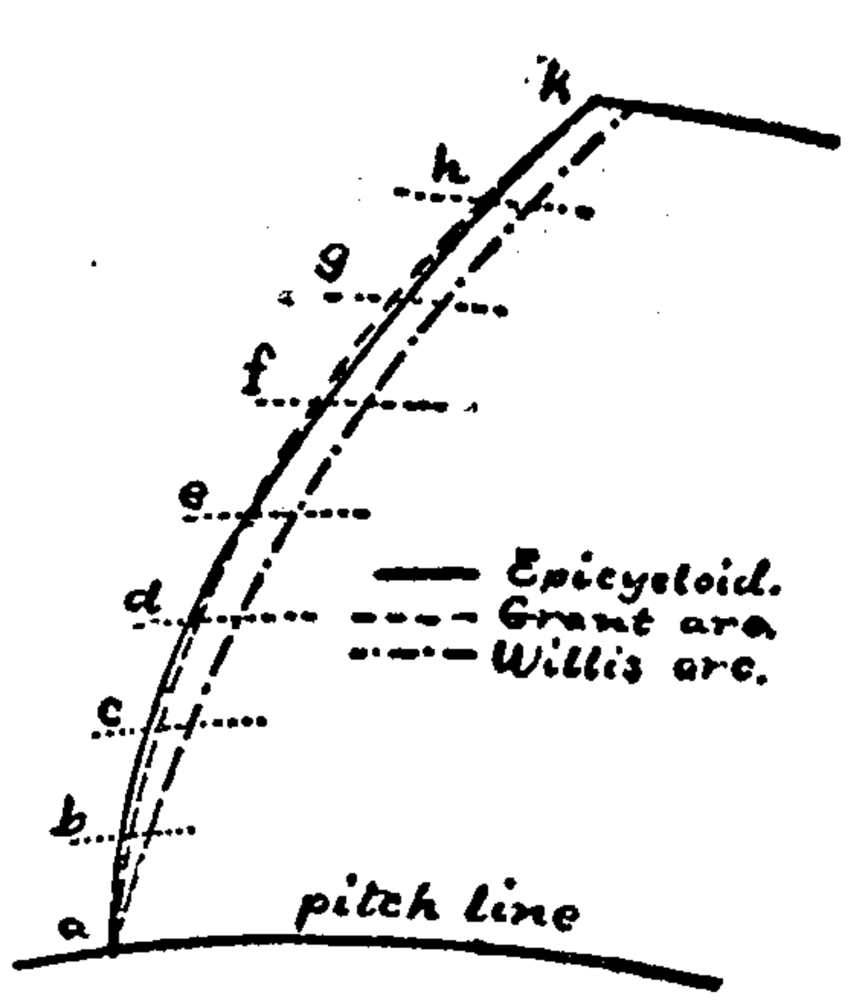


FIG. 9.

Figure 9 shows the true curve, and both the new and the Willis approximating arcs, the actual proportions being exagerated to show the errors more clearly.

The Willis arc runs altogether within the true curve, while the new arc crosses it twice.

We will take, for an example, the case of a twelve tooth pinion, which will show the errors at their greatest, and calculate them with great care for a tooth of three inch circular pitch, which is twice the size of the figure on page 13, and may be considered a very large tooth.

The distance from pitch line to addendum line is divided into eight equal spaces by parallel circles, and the distance along each circle, in ten thousandths of an inch, from the true curve to each odontographic arc, is as follows:

	GRANT.	WILLIS.	
At a	.0000	.0000	inches
" b	+.0088	+.0175	"
" c	+.0091	+.0244	"
" d	+.0056	+.0283	"
" e	.0000	+.0288	"
" f	-.0036	+.0297	"
" g	-.0061	+.0308	"
" h	-.0046	+.0342	"
" k	.0000	+.0397	"
Average,	.0042	.0260	"

It is seen that the new arc is in no place one hundredth of an inch in error, and that for a tooth of four pitch, a large size for cut work, its average error is one thousandth of an inch. A greater accuracy than this would be of no practical value.

The twelve tooth gear, for which the errors of both arcs were computed, shows them at their maximum value, for, as the number of teeth in the gear increases, the errors diminish, and for several locations their values for the new arc at c, which is the point of greatest error, are as follows:

For t = 12	c = .009 inches.
" 20	" .008 "
" 40	" .006 "
" 100	" .004 "
" 300	" .002 "

and the errors of the Willis arc are subject to the same rule.

The error of the Willis arc is plainly shown, at its greatest value, by the figure on page 13, where the dotted faces of the pinion teeth are correctly located by the Willis method.

To further test the accuracy of the new method, construct the same tooth face several times by the same process, using either the method by points, or the usual Willis process. Unless the work is most carefully performed, it will be found that the several results will not agree with each other by amounts that are noticeable, while by the new method they will be substantially the same curve.

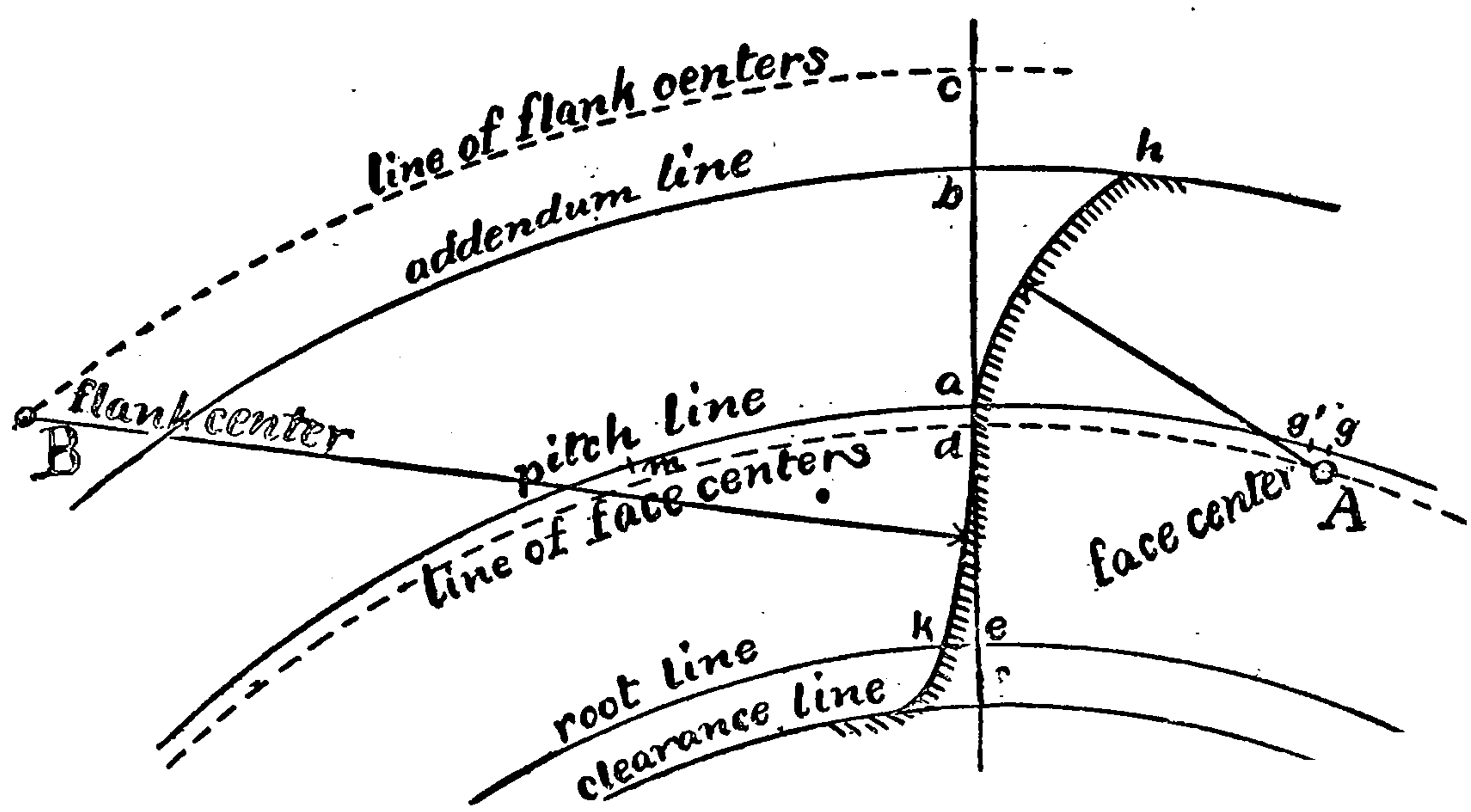
The new arc is most nearly correct at the most important point, the upper part of the curve, just where the Willis arc is most out of place, or where the true curve, unless drawn by some delicate and costly apparatus, in most likely to be out of place.

CIRCULAR AND DIAMETRAL PITCHES COMPARED.

CIR. P.	DM. P.
6	.52
5½	.58
5	.63
4½	.70
4	.78
3½	.90
3	1.05
2¾	1.15
2½	1.25
2¼	1.40
2	1.57
1¾	1.80
1½	2.10
1¼	2.50
1	3.14
¾	4.20
½	6.28

DM. P.	CIR. P.
½	6.28
¾	4.20
1	3.14
1¼	2 50
1½	2.10
1¾	1 80
2	1.57
2½	1.25
3	1.05
3½	.90
4	.78
5	.63
6	.52
7	.45
8	.39
9	.35
10	.31

THE NEW ODONTOGRAPH.



GENERAL DIRECTIONS.

Draw the pitch line and divide it for the pitch points m a g. Take from the tables, multiply or divide, as the case may require, by the pitch in use, and lay off, the addendum a b and a c, the clearance e f, the backlash g g′, the face distance a d, and the flank distance a c. Draw the addendum line through b, the root line through e, the clearance line through f, the line of face centers through d, and the line of flank centers through c. Set the dividers to the face radius, and draw all the faces a b from centers A. Set to the flank radius, and draw all the flanks a k from centers B. Round the flanks into the clearance line. The flanks of a gear of twelve teeth are straight radial lines.

ODONTOGRAPH TABLE.

EPICYCLOIDAL TEETH.

INTERCHANGEABLE SERIES.

From a Pinion of Twelve Teeth to a Rack.

Number of Teeth in the Gear.		For one Diametral Pitch. For any other pitch, divide by that pitch.		For one inch Circular Pitch. For any other pitch, multiply by that pitch.		
Exact.	Intervals.	Faces.	Flanks.	Faces. Rad.	Dis.	Flanks.
	12					
	13–14					
	15–16					
	17–18					
	19–21					
	22–24					
	25–29					
	30–36					
	37–48					
	49–72					
	73–144					
	145–rack.					

A PRACTICAL EXAMPLE

OF THE WORK OF THE NEW ODONTOGRAPH.

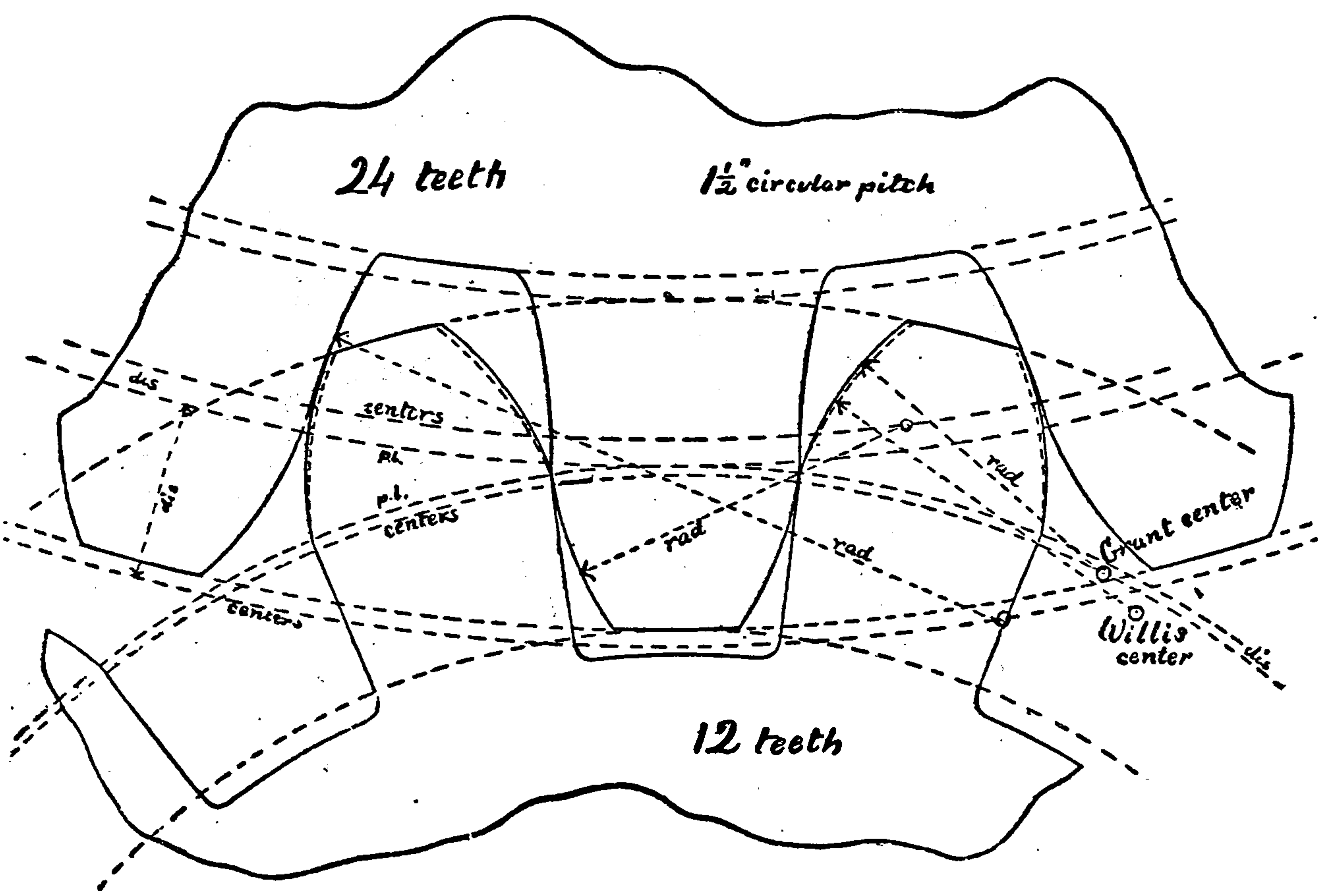


FIG. 10.

INTERCHANGEABLE SERIES.

EXAMPLE.—A gear of 24 teeth, and a gear of 12 teeth, of 1½ circular pitch.

DATA.—Take from the table the numbers to be used, which are as follows when multiplied by 1½.

For 24 teeth,	face rad, = 1.08	face dis, = .07.
" 24 "	flank " = 2.15	flank " = .54.
" 12 "	face " = .96	face " = .03.
" 12 "	flank " = ∞	flank " = ∞

Also take from the proper tables the pitch diameters 5.73 and 11.46 inches, the addendum, .5 inch, and clearance, .06 inch.

PROCESS.—Draw the two pitch lines, and divide for the pitch points. Draw the addendum, root, and clearance lines of both gears.

Draw the circles of centers, .03 inside the pitch line of the 12 tooth gear, and .07 inside of that of the other. Draw the circles of flank centers, .54 outside the pitch line of the 24 tooth gear, and draw straight radial flanks for the 12 tooth gear.

Draw the faces of the 12 tooth gear with the radius .96, and draw the faces of the 24 tooth gear with the radius, 1.08, and the flanks with the radius 2.15.

Round the flanks into the root line, and allow backlash by thinning the teeth according to judgement.

The dotted faces of the 12 tooth gear show them as they would be laid out by the Willis odontograph, and the figure also shows the two centers in place.

RADIAL FLANK SYSTEM.

TEETH NOT INTERCHANGEABLE.

Gears on this system must work together in pairs, each gear being fitted to its mate and to no other. See page 3. The process is the same that has been described on page 12 for the interchangeable set.

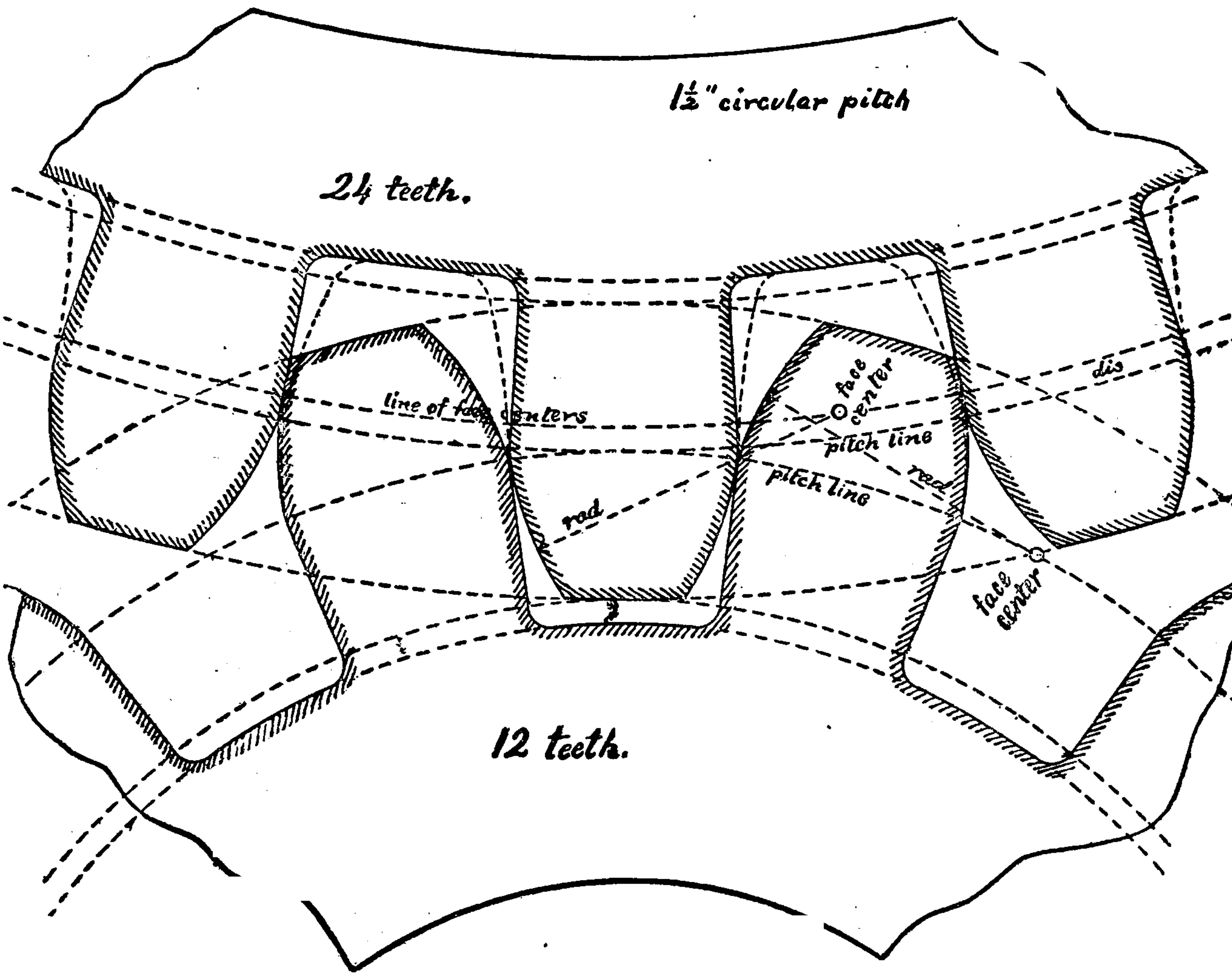


FIG. 11.

RADIAL FLANK SYSTEM.

EXPLANATION OF THE TABLE.—The upper number in each square is the face radius, the lower is the center distance.

The centers are mostly insid the pitch line, but some are on the line, and those having the negative sign are outside of it.

The tabular numbers are for one inch circular pitch, and must be multiplied by any other circular pitch in use. For the value for any diametral pitch, multiply the tabular number by 3.14, and then divide by the diametral pitch in use.

EXAMPLE.—A gear of 12 teeth, paired with a gear of 24 teeth. Circular pitch $1\frac{1}{2}$ inches.

DATA.—Take from the table for 12 teeth into 24, face radius =.68 and center distance = 0, and for 24 teeth into 12, radius = 72, and distance = .05. These multiplied by $1\frac{1}{2}$ give the values for use on the drawing, 12 rad. =1.02, 12 dis = 0, 24 rad. = 1.08, and 24 dis. = .07.

The addendum is one third the pitch, = $\frac{1}{2}$ inch, and the proper tables give the clearance =.06, and the pitch diameters = 5.73 and 11.46 inches.

PROCESS.—Draw the two pitch lines 5.73 and 11.46 inches in diameter and space them for the teeth.

Lay off the addendum, .5 inch, and the clearance, .06 inch, and draw the addendum, root, and clearance lines.

Draw all the faces of the twelve tooth gear, from centers on its pitch line, with the radius 1.02. Draw all the faces of the 24 tooth gear from centers on a line .07 inch inside its pitch line, with the radius 1.08 inches. Draw straight radial lines for the flanks of all the teeth.

THE INVOLUTE TOOTH.

With the exception of the epicycloid, the only curve in extensive use for the working face of a gear tooth, is the involute.

THE INVOLUTE CURVE.

As the rolling circle A of fig. 3 increases in size, it finally, when of infinite diameter, becomes the straight line d g of fig. 15, while the epicycloid traced by a fixed point in the circle becomes the involute.

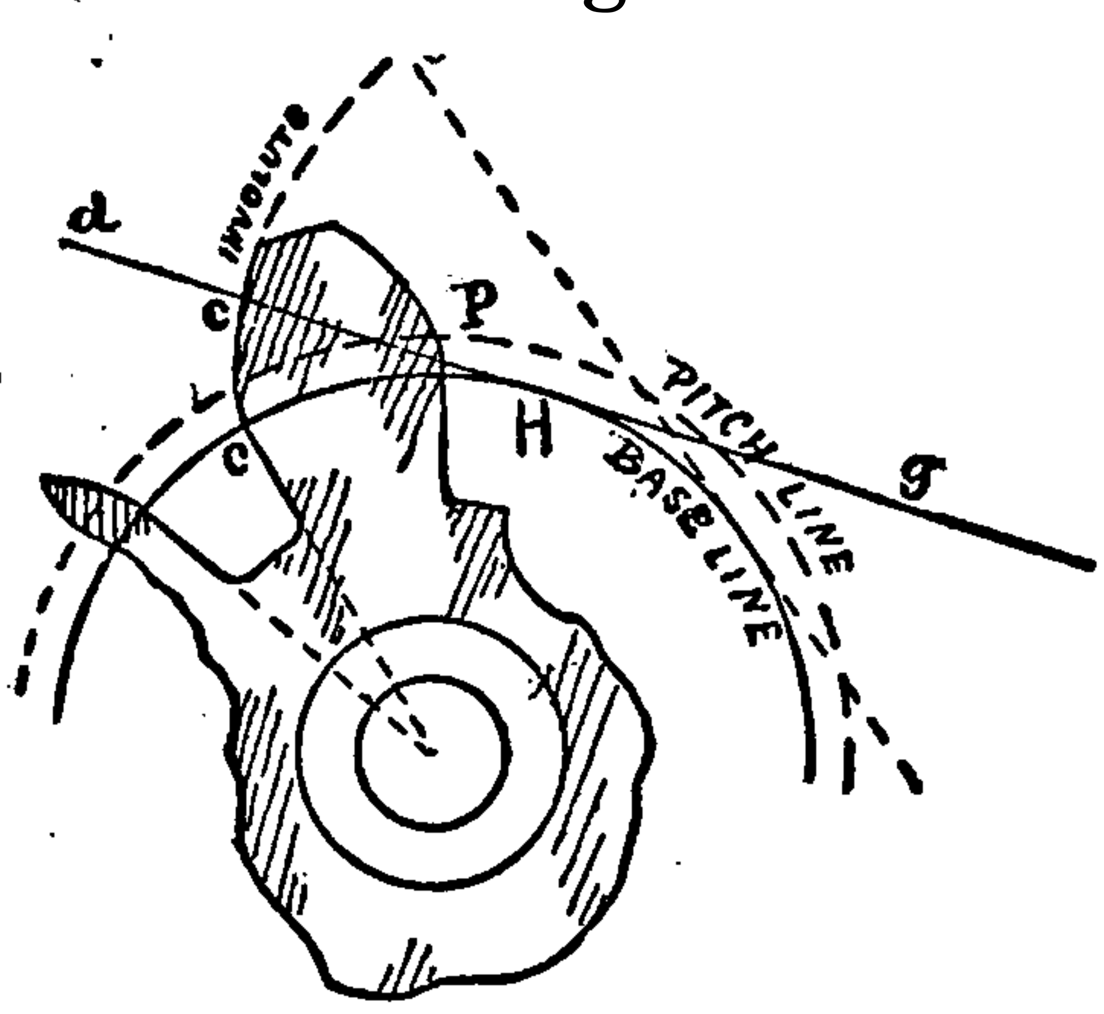


FIG. 15. THE INVOLUTE.

The involute is, therefore, not a new or separate curve, but simply a particular case of the epicycloid. It is the infinite form of the epicycloid.

As the rolling circle of infinite diameter is the same thing as a straight line, the involute can be formed by a fixed tracing point in a cord which is unwound from a circle, called its "base circle," which has been wrapped or "involved" in it, and from this property it derives its name.

ITS UNIFORM ACTION.

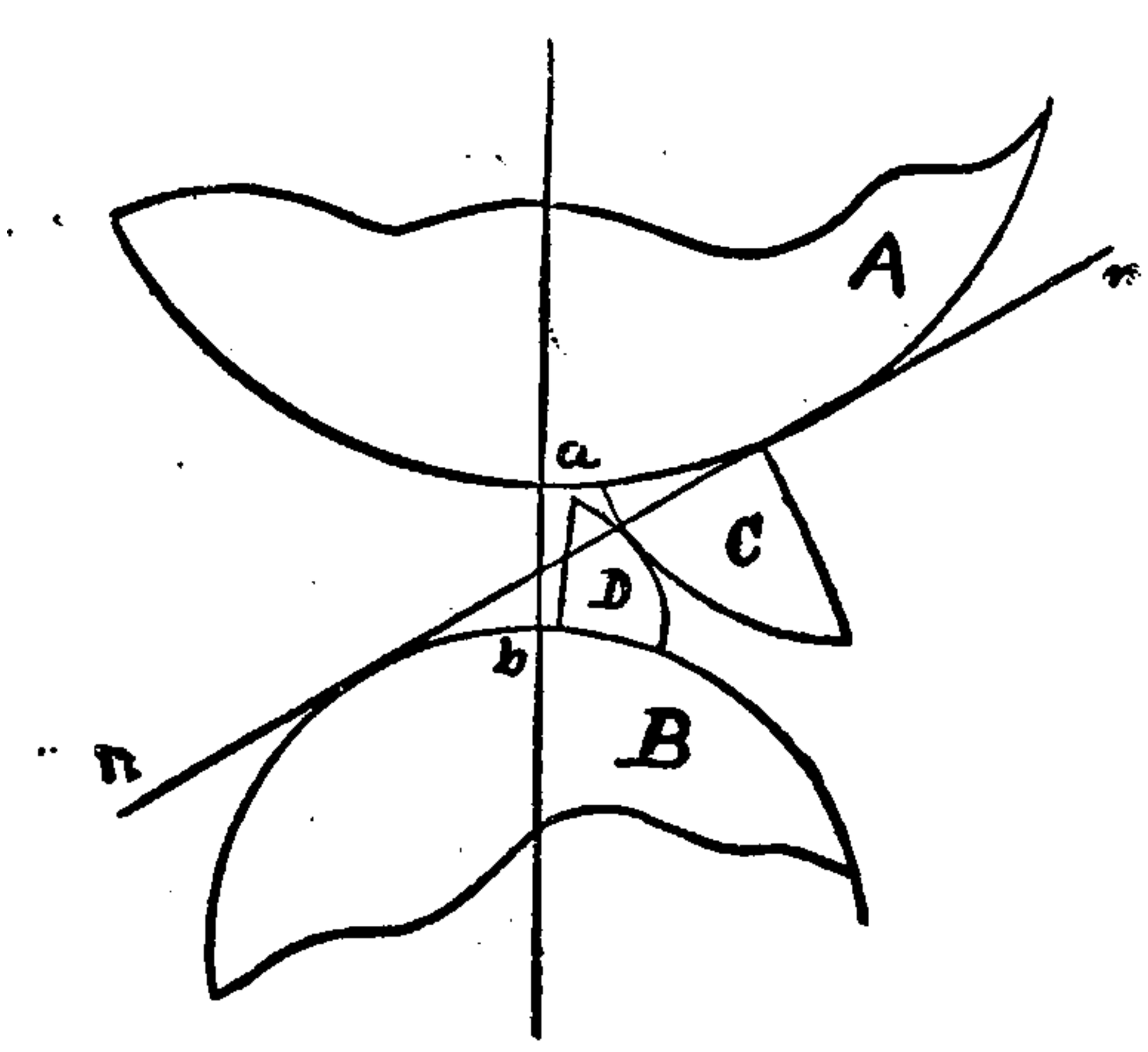


FIG. 16. EXTERNAL EPICYCLOIDS.

If the two circles A and B, fig. 16, are separated by the distance a b, and work together by means of two external epicycloids C and D, the motion communicated will be irregular, for the conditions of uniformity are that the two circles shall touch, and that the external curve of one shall work with the internal curve of the other. See page 2 and figure 4.

The amount of this irregularity will depend on the proportion between the separating distance a b and the diameter of the rolling circle which describes the epicycloids. If the proportion is very small, the irregularity will be very small, and if the rolling circle has an infinitely great diameter, the proportion and the irregularity will be infinitely small, that is, zero. Therefore, involutes will work together with perfect regularity and are suitable curves for gear teeth.

ITS ADJUSTIBILITY.

If the rolling circle is infinitely large, the proportion between the separating distance and it will always be zero, and it will not be altered by any finite alteration of the former, and therefore the uniformity of the action of involute teeth is not in any way dependent upon, or affected by any change of the separating distance. The action will be perfect as long as the curves remain in contact, and this is a property of the greatest practical value, which gives the involute a great advantage over every other known or possible curve.

The curve of any gear tooth must of necessity be a "rolled curve" formed by a fixed object attached to the plane of or moving with some curve that rolls upon the base curve of the tooth, and, as the involute is the infinite form of any rolled curve, it is the only form that can possess this property of adjustibility.

ITS UNIFORM PRESSURE AND FRICTION.

The point of contact of the two involutes C and D will always be upon the straight line of action m n, the common tangent of the two base circles, commencing at its point of tangency with one circle, and ending at the same point on the other.

The direct pressure between the two teeth will always be in the direction of the line of action, and uniform both in direction and in amount, a property that is peculiar to the involute curve, and which contributes greatly to the smooth action and even wear of involute teeth. Friction is substantially in proportion to direct pressure, and when the pressure is uniform, the friction will be uniform, and no part of the curve will be more likely to wear away than any other part. The durability of a tooth, particularly when doing heavy work, depends on the uniformity of the friction as well as upon its absolute amount.

THEORETICAL CONSTRUCTION.

To draw the involute curve through the pitch point a of two pitch circles A and B, draw the line of action m n at any desired angle with the line of centers, usually 75°, and then draw the base circles C and D, touching the line of action at e and d, where the perpendicular radial lines e g and f d meet it. From a, step off any number of short steps along the line of action and around the base line to any point s, then draw any number of tangent lines b c, t v, then step off the distances s b c, s t v, s b, etc., each equal to s d a, and the points c, v, b, etc., will be points of the curve. Any line, as w c x drawn through c at right angles to h c, will be tangent to the curve. The working part of the curve must not be extended beyond the circle k e p through the point of contact of the line of action m n and the base line C, for beyond that point it will interfere with the radial flank of the tooth it works with.

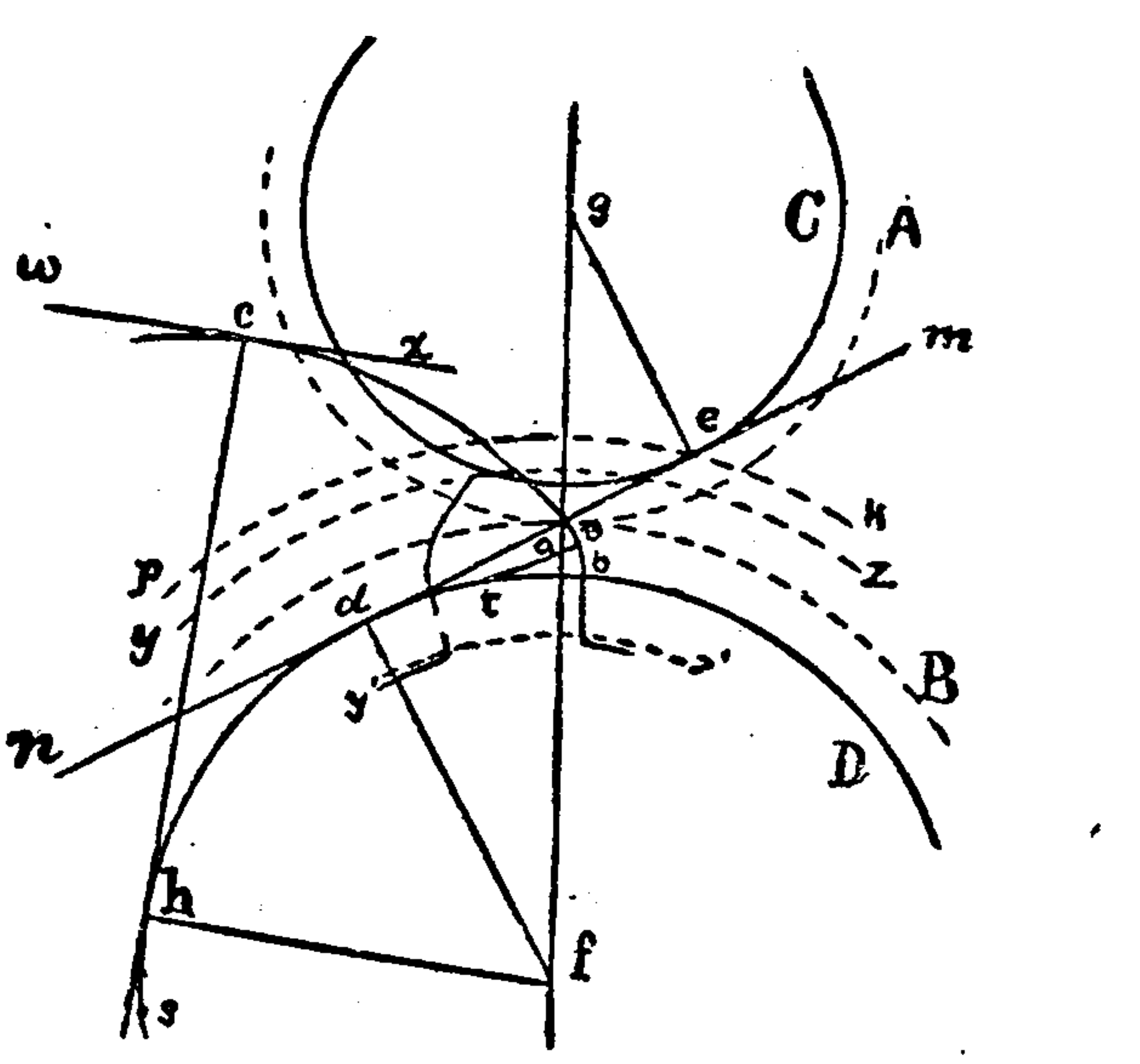


FIG. 17. CONSTRUCTION.

The curve is generally limited by the addendum line z y, at an arbitrary distance from the pitch line B, and ends at b on the base line D, where it is perpendicular to the base line. It is continued within the base line by a radial line as far as the root line z y, and is then rounded into the clearance line.

The matter under epicycloidal teeth, pages 3, 4, and 5, regarding the pitch, addendum, clearance, and backlash, will apply as well to involute teeth.

ANGLE OF ACTION.

The angle m a g may be less, but not greater, than the value found from the formula

$$m\,a\,g = 90^\circ - \frac{180^\circ}{s}$$

in which s is the number of teeth in the smallest gear in the pair. If the angle is greater than this the motion will not be continuous, as each pinion tooth will pass out of action before the next one is in position to act.

INTERCHANGEABLE SETS.

Any number of involute gears from base circles of different diameters will work together correctly and interchangeably if all are of the same pitch, and have the same angle of action.

If we put s = 12 teeth, we find

$$m\,a\,g = 90^\circ - \frac{180^\circ}{12} = 75^\circ$$

the value for the common twelve to rack interchangeable set, and if we use fifteen as the smallest number of teeth in the set, we have an angle of action of 78°.

PRACTICAL CONSTRUCTION.

When the involute is to be brought into use, we meet with the same difficulties as with the epicycloid, for its theoretically correct construction is not easily and accurately accomplished, and we must adopt some short cut of approximative accuracy.

The principle of the epicycloidal engine of fig. 7 may be applied to the construction of the involute, the ribbon s being drawn tight and straight as it is unwound from the base circle, but the same difficulties prevent its use for ordinary purposes.

THE OLD RULE.

A defective rule in common use draws the whole curve from base line to addendum line, as one circular arc. The angle m a g is laid off at 75°, sometimes at 75½°, the distance a c is made equal to one quarter of the pitch radius a g, and the tooth curve is drawn from c as a center.

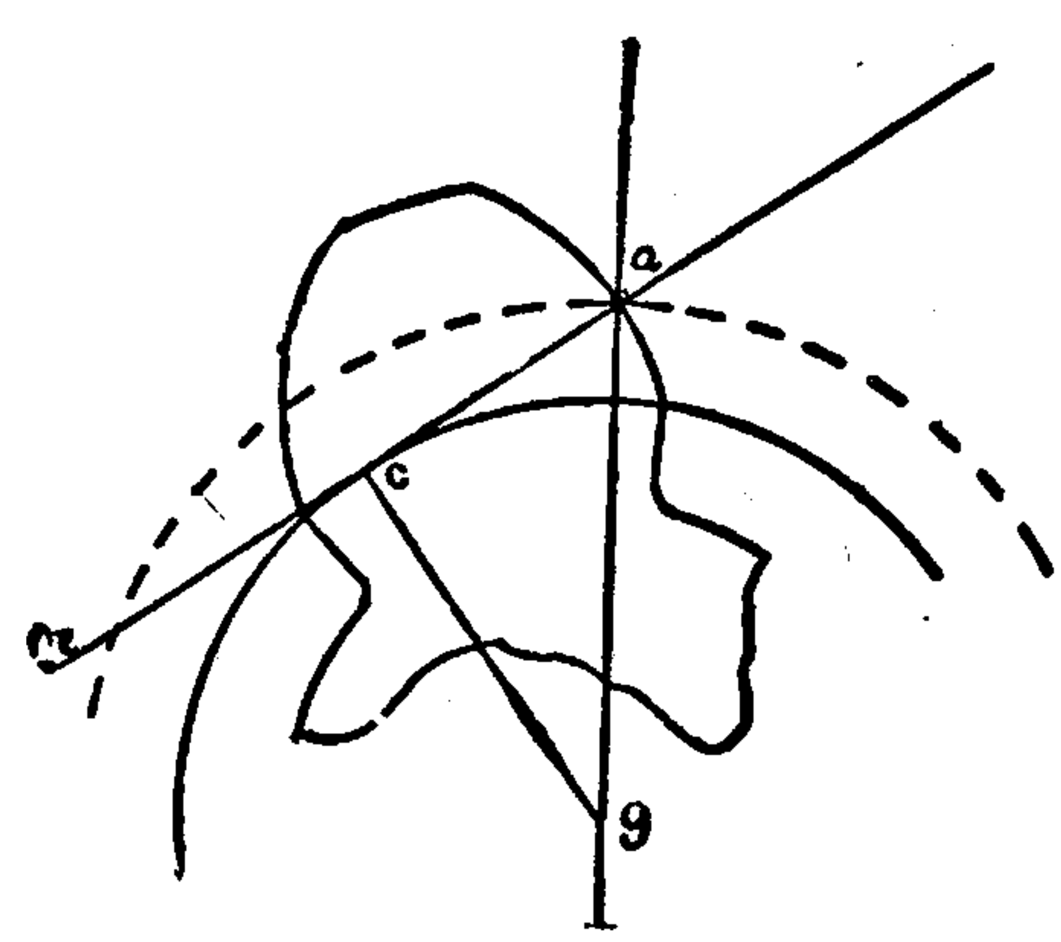


FIG. 18. THE OLD RULE.

This rule is simple, to be sure, but it gives the faces shown by the dotted lines of the figure on page 22, and is abominably wrong and worthless.

If it would round off the points of the teeth of a large gear, it would be useful to correct interference, but it greatly rounds the teeth of a small gear that needs little or no correction, and gives the curve on a large gear in nearly its theoretical position, without the allowance for interference that must be made.

It is not to be wondered that the involute tooth is in small favor with practical mechanics who use this bungling method, and who do not understand that the trouble is not in the involute system, but in its defective application.

A NEW METHOD.

In devising a method for drafting the involute tooth, I have borne in mind that a minute degree of accuracy is not the essential requirement, for although substantial accuracy must be secured, simplicity and convenience are qualities that must also be considered.

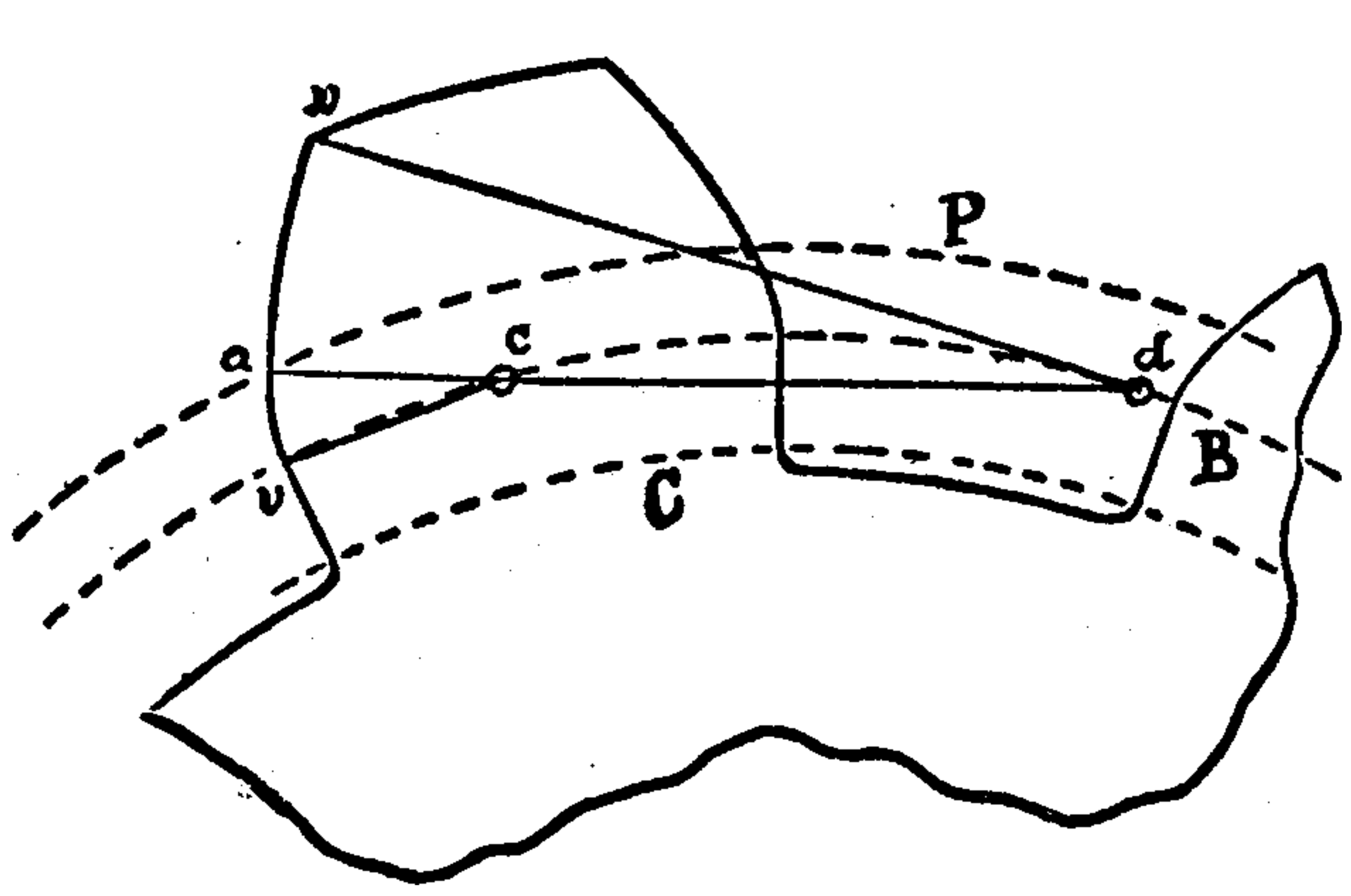


FIG. 19. THE NEW METHOD.

The method, in general terms, and given in full on pages 22 and 23, is to give, by a table, the distance of the base circle B, see fig. 19, inside the pitch circle P, and to give by the same table, the distances or radii a c and a d from the pitch point a to centers c and d on the base line. The face arc a w is drawn from the center d and the flank arc a v from the center c.

The table, page 23, is for one diametral pitch, and covers the common twelve to rack interchangeable set. Interference must be corrected, when necessary, as explained below in detail.

INTERFERENCE.

As indicated above, the involute face will interfere with the radial flank of the mating tooth if the addendum is greater than a certain amount, and as the addendum in common use for the interchangeable set generally exceeds this limit, we must gererally make corrections to avoid this trouble.

Figure 20 shows the interference, its effect, and its correction.

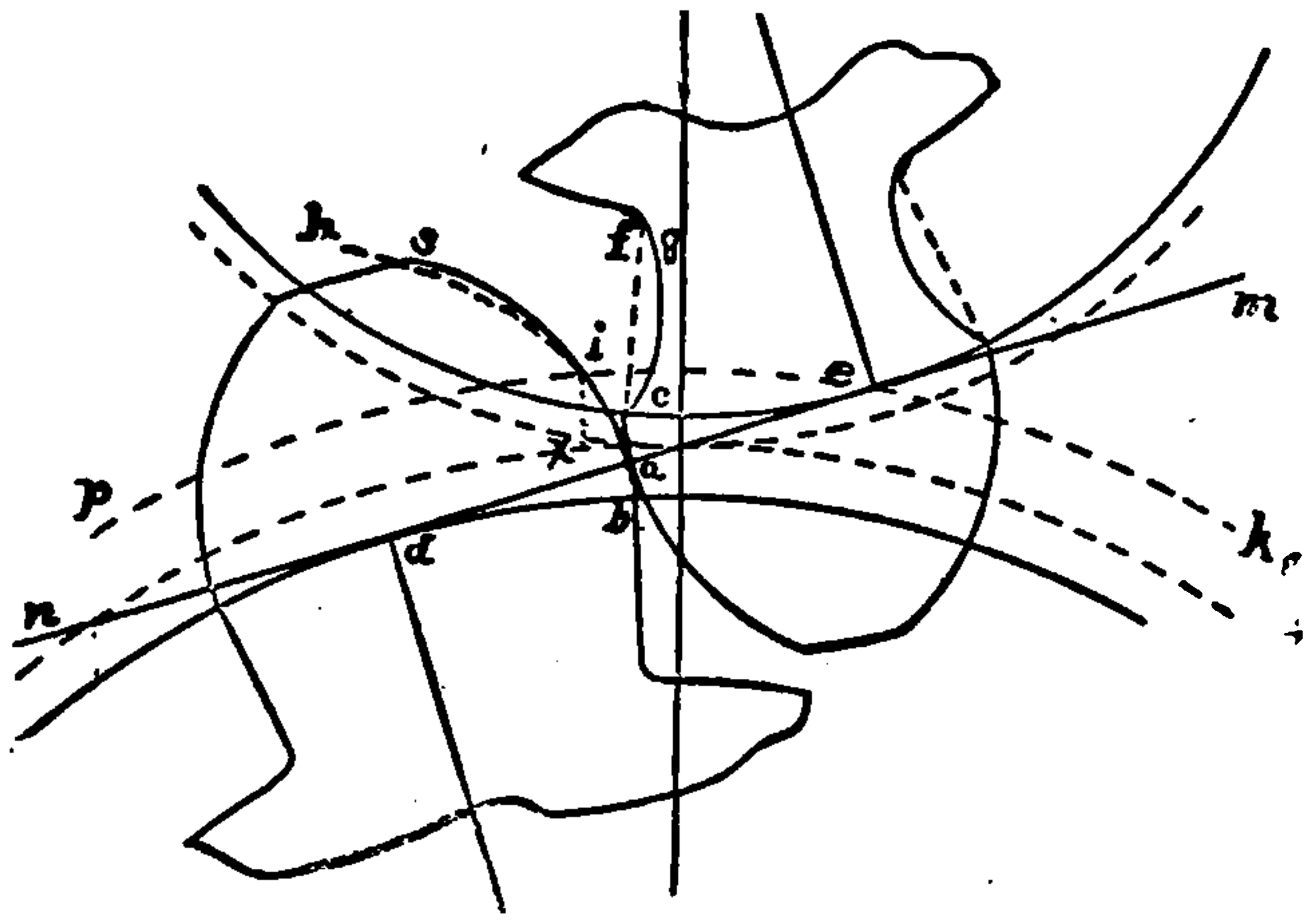


FIG. 20. INTERFERENCE.

The working face of the involute should be limited at i by the circle k p through the tangent point e, but if the usual addendum continues it beyond that line, to s, the extension s i will interfere with the radial flank c f, and the uniformity of the action will be destroyed.

To correct it we must either weaken and spoil the shape of the mate tooth by undercutting the flank c f by an epitrochoidal line c g, or we may, and much better, round off the point of the tooth by an epicycloidal curve i h.

The amount of this interference will depend on, and increases with, the angle of action, and also depends upon the number of teeth in each gear. It is greatest on a large gear or rack that runs in a small pinion, and least on a pinion running in a large gear. When the angle of action is 75° there is no interference when both gears of a pair have thirty or more teeth, or when an equal pair have twenty-one or more teeth. When one gear has more, and the other has less than thirty teeth, the larger may need correction, but the smaller never will.

Interference Table

For one diametral pitch and 3 1-7 inch circular pitch. Angle of action, 75°.

Number of Teeth in the Mate.

Number of Teeth in the Gear Being Drawn.	12	13 14	15 16	17 18	19 21
12	.58 .01	.67 .01			
13–14	.56 .02	.66 .01			
15–16	.54 .02	.64 .01	.75 .01		
17–18	.53 .02	.62 .02	.72 .01		
19–21	.51 .02	.60 .02	.69 .01		
22–24	.50 .02	.58 .02	.67 .01		
25–29	.49 .03	.57 .02	.65 .02	.75 .01	
30–36	.47 .03	.55 .02	.63 .02	.72 .01	
37–48	.45 .03	.53 .02	.61 .02	.69 .01	
49–72	.44 .04	.52 .03	.59 .02	.66 .02	.73 .01
73–144	.42 .05	.49 .04	.56 .03	.63 .02	.70 .01
145–∞	.40 .06	.46 .05	.53 .04	.60 .02	.67 .01

The amount of the interference, the correction to be made by rounding off the point of the tooth, is very small and may generally be neglected on small pinions.

It is given by the lower figures in the table, which shows that it is never more than a sixteenth of an inch on a large tooth of one diametral, or three inch circular pitch, and not over two or three hundredths of an inch on a gear of that pitch having few teeth. The table also shows by the upper figures the limit point or distance i x above the pitch line where the interference commences.

The tabular numbers must be divided by the diametral pitch that may be in use, and for any circular pitch it is sufficient to divide the tabular number by 3 and then multiply by the pitch.

The table takes no notice of an interference of less than a hundredth of an inch on a tooth of three inch circular pitch.

When, as is usually and should always be the case, the gear being drawn belongs to the twelve to rack interchangeable set, the interference should be computed for a mate gear of twelve teeth, or by the first vertical column of the table. In this case the error will not be perceptible if the limit distance to point of first interference be always assumed to be half the addendum.

When the work is upon a rough cog wheel or mill gear, or upon a pattern for a cast gear, the only correction needed for interference, is a slight rounding off of the points if it is a rack or very large gear, and a mere touch on the point of a gear of few teeth.

EPICYCLOIDAL vs. INVOLUTE TEETH.

A COMPARISON.

The epicycloidal tooth is in much greater use and favor than the involute form, particularly for heavy work, both writers and mechanics generally preferring it, and seldom giving the preference to its rival. It is difficult to account for this favor except, as in the case of the circular pitch system, on the ground that the epicycloid was adopted in the infancy of mechanical science, and holds its place by virtue of prior possession, for the involute has certainly the advantage from every practical point of view.

Space will not permit an extended discussion with the necessarily bulky demonstrations, but, if the two curves be closely and carefully examined under the same conditions within the limits of either the twelve tooth or the fifteen or higher tooth interchangeable series, with the customary addendum, which limitation will cover nine-tenths of the gears in actual use, it will be found that they compare as follows:

I. Adjustibility. Involute teeth alone can possess the remarkable and practically invaluable property, that they are not confined to any fixed radial position with respect to each other, for, as long as one pair of teeth remains in action until the next pair is in position, the perfect uniformity of the action of the curve is not impaired.

The shafts may be at the proper distance apart, or not, as happens, and they may change position by wearing, or variably as when used on rolls, or may be forced together to abolish backlash, and, in fact, the curve is wonderfully adapted to the variable demands, and will accommodate itself to errors and defects that cannot be avoided in practice.

Epicycloidal teeth must be put exactly in place and kept there, and the least variation in position, from bad workmanship in mounting, or by wear or alteration of the bearings in use, will destroy the uniformity of the motion they transmit. When perfectly mounted and carefully kept in order, epicycloidal teeth are as good as any in this respect, but for most practical purposes they are decidedly inferior.

This virtue of the involute is always recognized by writers, but is seldom given the position its importance demands, for it is only as a result of experience in making and using gears, that its importance can be seen at its full value.

II. Uniformity. The direct force exerted by involute teeth on each other, is exactly uniform, both in direction and in amount, and this property ensures *a uniform wearing action of the teeth*, a nearly uniform thrust on the shaft bearings, and a steadiness and smoothness of action that cannot be claimed for epicycloidal teeth under any circumstances.

The direct pressure acting between epicycloidal teeth is variable in amount and very variable in direction, and consequently the friction and wearing action between the teeth, as well as the thrust on the bearings, is variable between wide limits.

III. Friction. The measure, for purposes of comparison, of the loss of power by friction, is the product of the direct pressure between the teeth, multiplied by their rate of sliding motion on each other.

This measure is always in favor of the involute by a decided advantage, although the advantage is usually claimed for the epicycloid, both as to maximum values and average values, and as this is an important point, it should have great weight in deciding between the two forms of teeth, for the element of friction is of chief importance in determining the life of a gear in continual and heavy service.

The epicycloid is mostly in use for heavy gearing from a mistaken view of this point, it being generally taught that its friction is the least.

IV. Thrust on Bearings. Here the advantage is with the epicycloidal tooth, but not by a large amount, and not in a matter of first consequence.

The thrust on the bearings due to the action of the teeth on each other is but a fraction of the whole thrust due to the power being carried, and as the average thrust of the teeth is but little in favor of the epicycloid, and as the maximum thrust is always from that form of tooth, the two forms may be said to be well balanced in this respect. Moreover, the thrust of the involute is but slightly variable, while that of the epicycloid varies from large values at the points of first and final action to nothing at all at the line of centers, and must give rise to a rattling and uneven action.

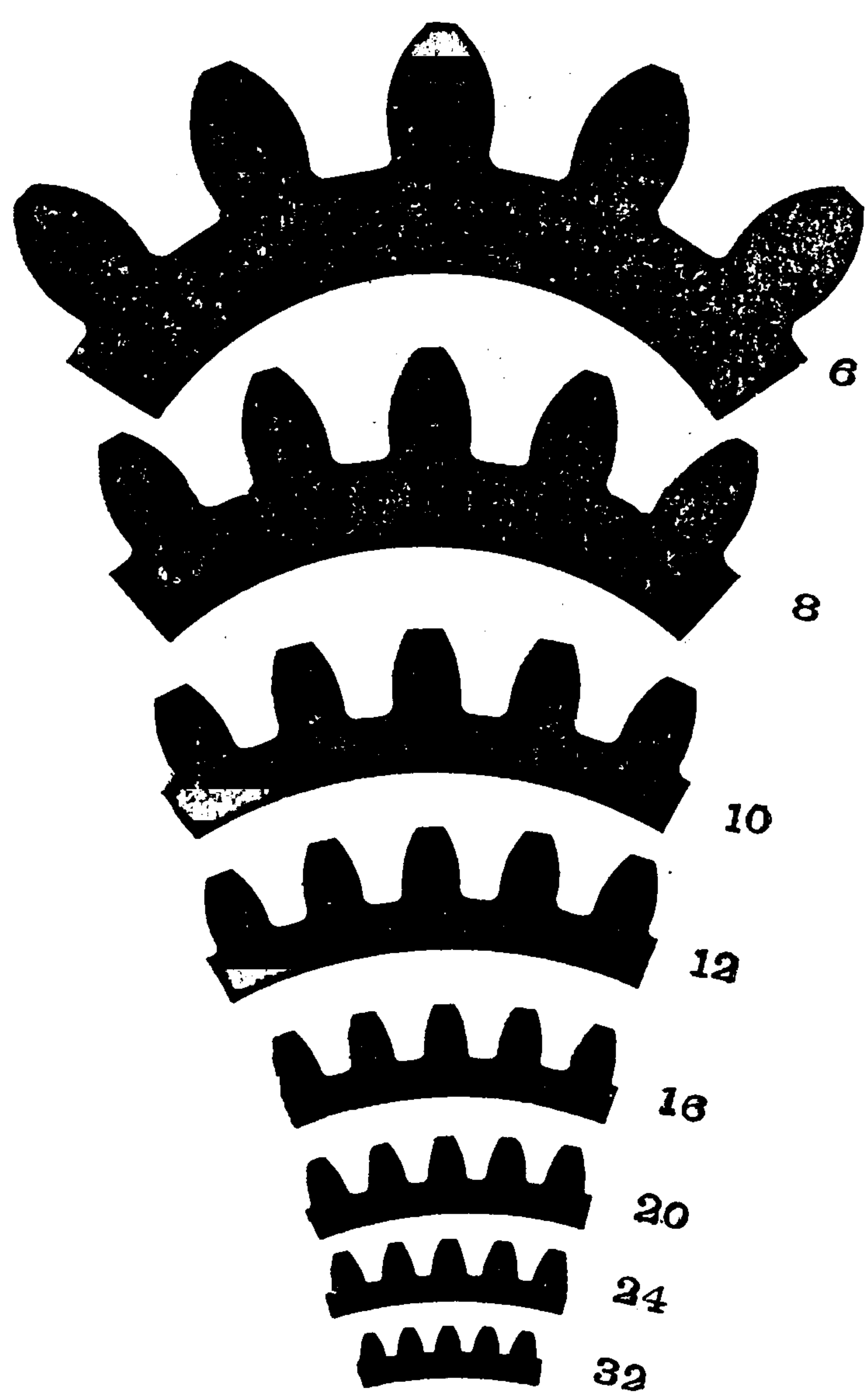
V. Strength. The weakest part of a tooth is at its root, and as the involute tooth spreads more than the epicycloidal tooth, it is stronger at that point and has a considerable advantage.

VI. Appearance. This is a small point and a matter of opinion, but is worth mention. The involute is a simple and graceful single curve, while the epicycloid is a double and not mechanically a neat curve, and, *as generally drawn*, has a decided bulge or even a plain corner where the two halves join at the pitch line.

In General. As the involute has the advantage of the epicycloid, in nine actual cases out of ten, with respect to adjustibility in position, in uniformity of wear and action, in loss of power and change of shape by friction, in strength. and in appearance, and is but a shade, if any, inferior with regard to the thrust on the bearings, it may be, and should be accorded first place for any and every practical purpose. The writer can imagine no possible case, unless it be in connection with a pinion of very few teeth, where the epicycloid would have either a theoretical or a practical advantage over the involute.

INVOLUTE TEETH

OF SEVERAL DIAMETRAL PITCHES.



ODONTOGRAPH TABLE.

INVOLUTE TEETH.

INTERCHANGEABLE SERIES.

FROM A PINION OF TWELVE TEETH TO A RACK.

NUMBER OF TEETH IN THE GEAR.		FOR ONE DIAMETRAL PITCH. For any other pitch, divide by that pitch.			FOR ONE INCH CIRCULAR PITCH. For any other pitch, multiply by that pitch.		
Exact.	Intervals.	Base Distance.	Face Radius.	Flank Radius.	Base Distance.	Face Radius.	Flank Radius.
12	12	.20	2.70	.83	.06	.86	.27
13	13	.22	2.87	.93	.07	.91	.30
14	14	.23	3.00	1.02	.07	.95	.33
15	15	.25	3.15	1.12	.08	1.00	.36
16	16	.27	3.29	1.22	.08	1.05	.40
17	17	.28	3.45	1.31	.09	1.09	.43
18	18	.30	3.59	1.41	.09	1.14	.46
19	19	.32	3.71	1.53	.10	1.18	.50
20	20	.33	3.86	1.62	.10	1.22	.53
21	21	.35	4.00	1.73	.11	1.27	.57
22	22	.37	4.14	1.83	.11	1.32	.60
23	23	.39	4.27	1.94	.12	1.36	.63
25	24–26	.42	4.56	2.15	.13	1.45	.70
28	27–29	.45	4.82	2.37	.14	1.54	.77
31	30–32	.50	5.23	2.69	.15	1.67	.88
34	33–36	.57	5.77	3.13	.17	1.84	1.00
38	37–41	.63	6.30	3.58	.19	2.01	1.16
44	42–48	.73	7.08	4.27	.22	2.26	1.38
52	49–58	.87	8.13	5.20	.26	2.59	1.70
64	59–72	1.07	9.68	6.64	.32	3.09	2.18
83	73–96	1.39	12.11	8.93	.42	3.87	2.90
115	97–144	1.92	16.18	12.80	.58	5.16	4.15
192	145–288	3.20	25.86	22.30	.96	8.26	7.30
576	289–rack	9.60	73.95	70.10	2.88	23.65	22.30

INTERFERENCE

FOR TWELVE TO RACK INTERCHANGEABLE SET.

Teeth In the gear.	12	13 14	15 16	17 18	19 21	22 24	25 29	30 36	37 48	49 72	73 144	145 ∞
Pitch.	Amount of the Interference.											
One in. cir.	.003	.007	.007	.007	.007	.007	.010	.010	.010	.013	.017	.020
One diamet'l	.01	.02	.02	.02	.02	.02	.03	.03	.03	.04	.05	.06

Interference always to commence at a point half way between pitch line and addendum line.

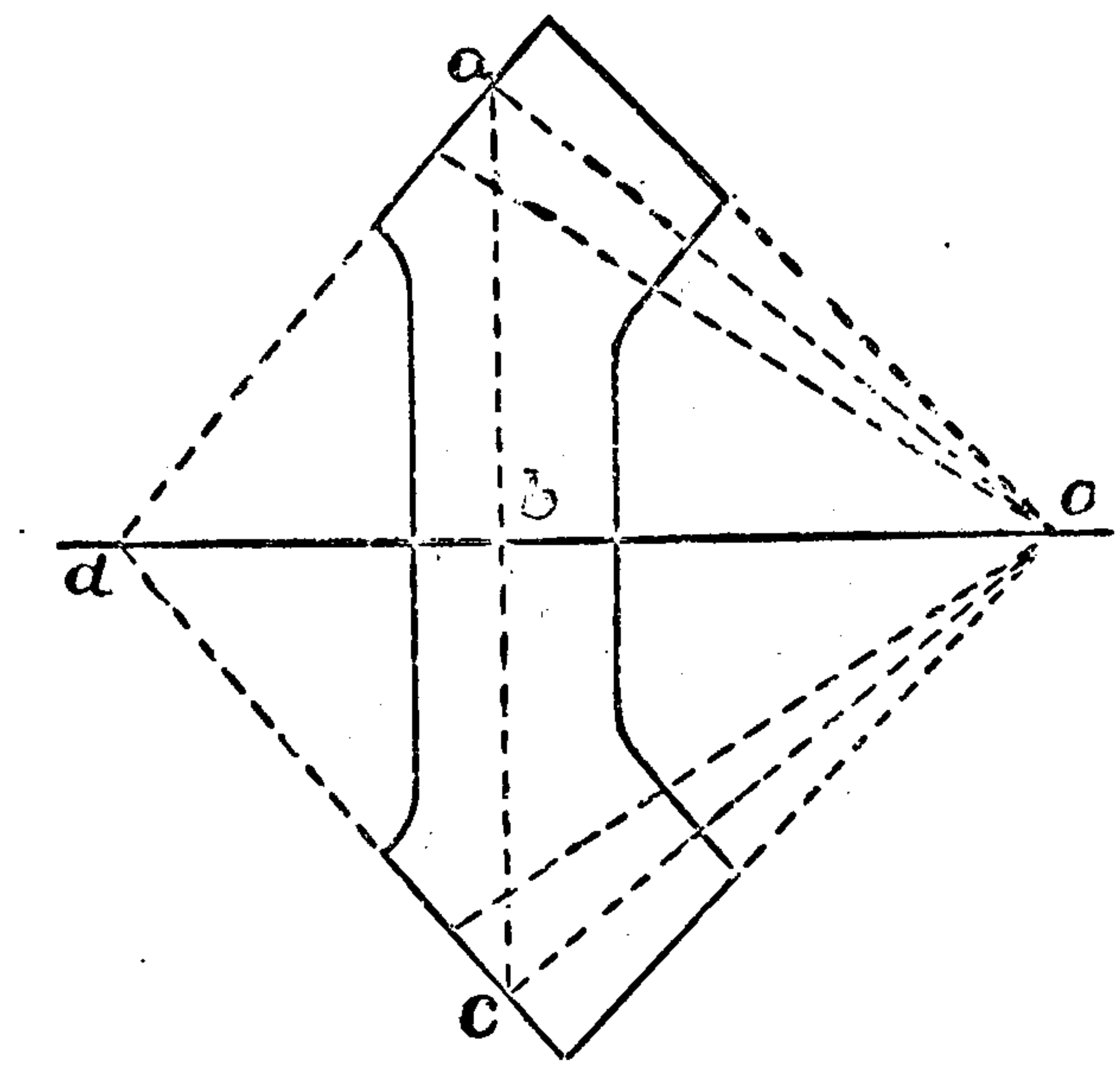


FIG. 24.

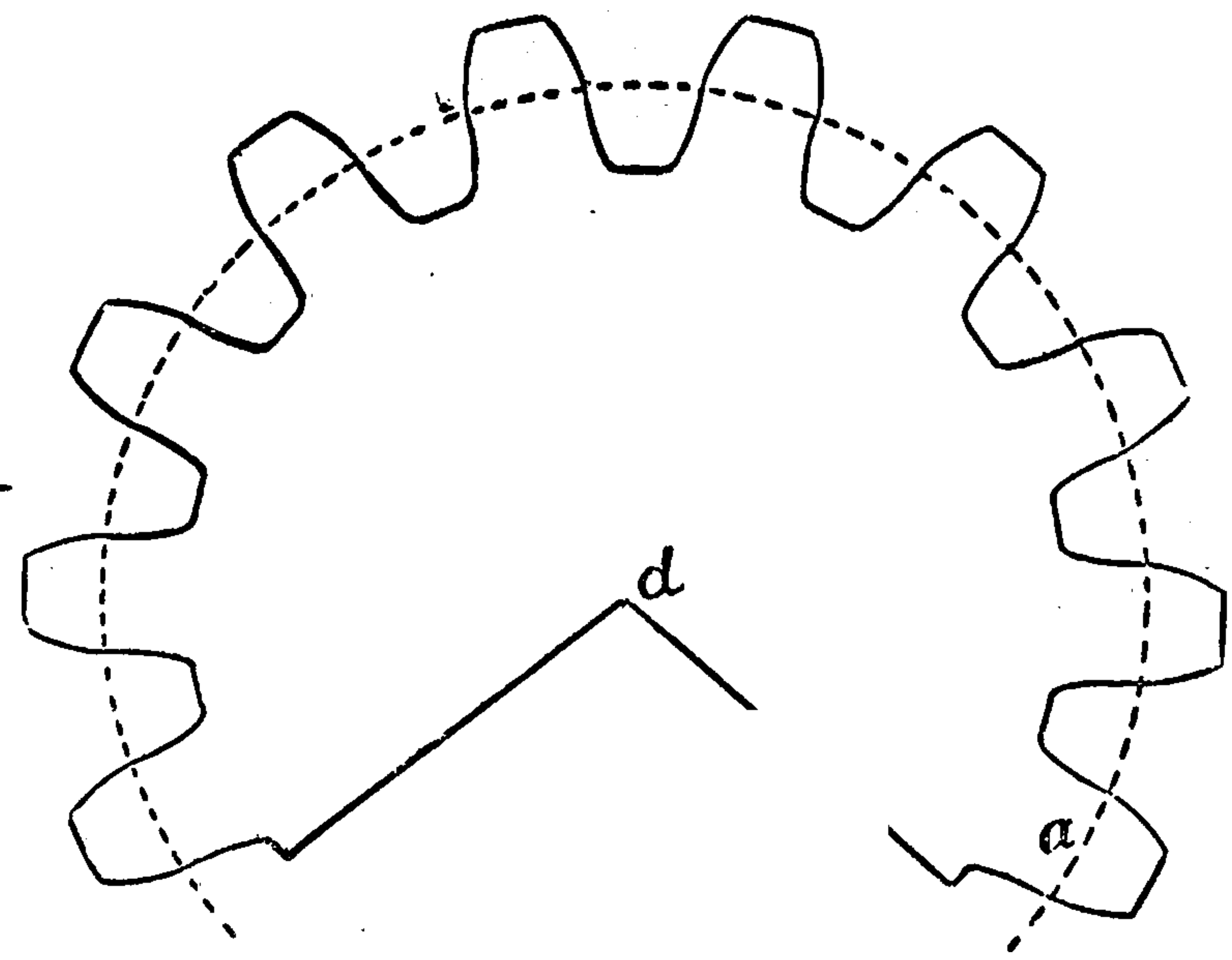


FIG. 25.

BEVEL GEARS.

In laying out the teeth of a bevel gear but one new point needs to be considered. The working pitch diameter a b c is not to be used, but the teeth are to be drawn on the conical pitch diameter a d c, developed or rolled out as in fig. 25.

The conical diameter a d c may be found from a drawing, or if the gears are of some common proportion, from the following table by multiplying the true pitch diameters by the tabular numbers given for that proportion.

TABLE OF CONICAL PITCH DIAMETERS OF BEVEL GEARS.

Proportion.	Larger Gear.	Smaller Gear.
1 to 1	1.41	1.41
2 " 1	2.24	1.12
3 " 2	1.80	1.20
3 " 1	3.16	1.05
4 " 3	1.67	1.25
4 " 1	4.12	1.03
5 " 4	1.60	1.28
5 " 3	1.94	1.17
5 " 2	2.69	1.08
5 " 1	5.10	1.02
6 " 5	1.56	1.30
6 " 1	6.08	1.01
7 " 1	7.07	1.01
8 " 1	8.06	1.01
9 " 1	9.06	1.01
10 " 1	10.05	1.01

EXAMPLES.—A miter gear, proportion 1 to 1, of 4 pitch, 6″ diameter, and 24 teeth, has a conical diameter of 6″ x 1.41 = 8.46″; and there are 24 x 1.41 = 33.8 teeth on the full circle of the developed cone.

A pair of bevel gears of 3 to 1 proportion, 48″ and 16″ diameters, 36 and 12 teeth, have conical diameters 48″ x 3.16 = 151.68″, and 16″ x 3.16 = 16.80″, and there are 36 x 3.16 = 113.76, and 12 x 1.05 = 12.60 teeth on the full circles of the developed cones.

105

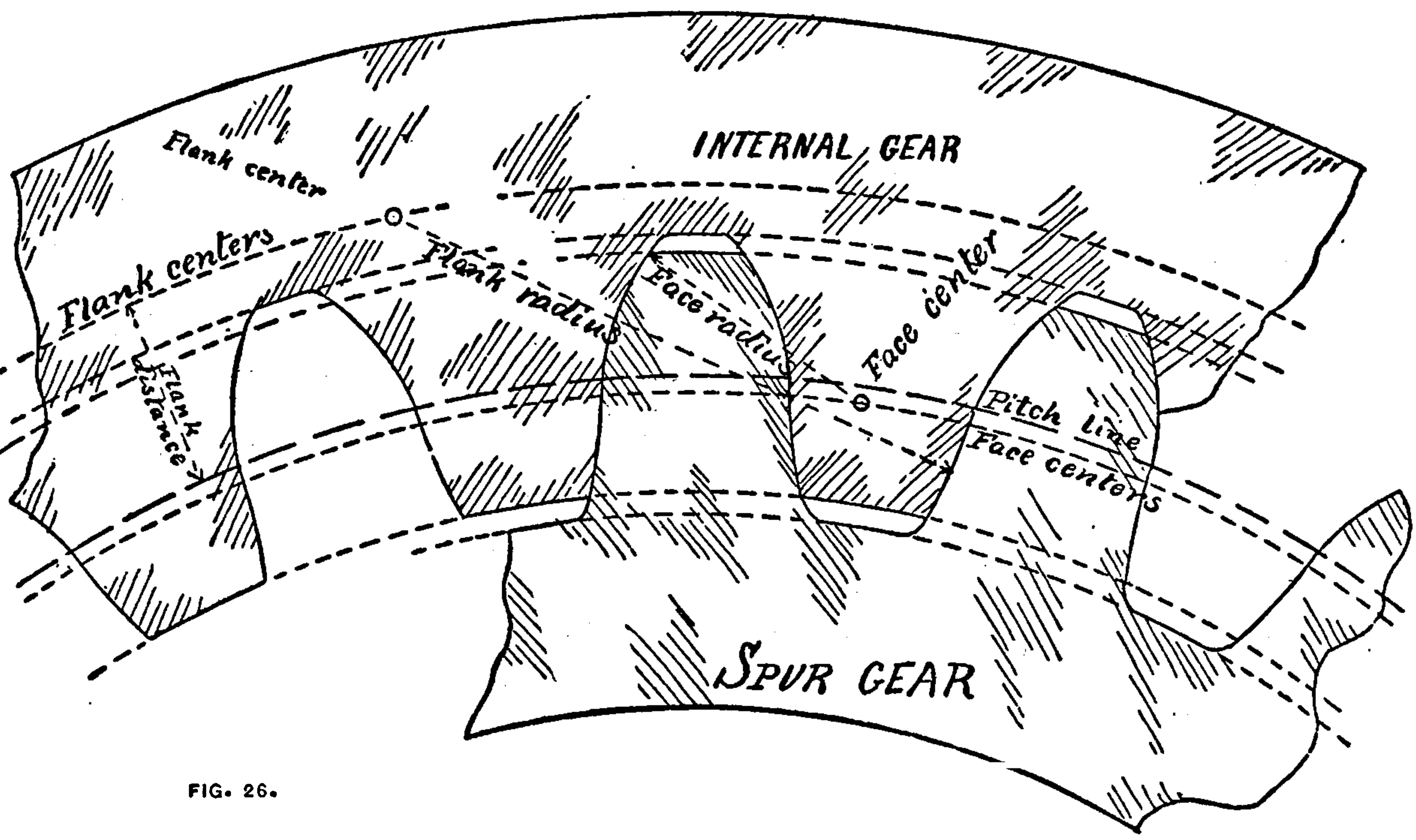


FIG. 26.

INTERNAL GEARS.

The internal gear, sometimes called the "annular" gear, is drawn by the rules for spur gears, the teeth of a spur gear being the spaces between the teeth of an internal gear of the same pitch diameter, with the backlash and clearance reversed in position.

Involute teeth should end at the base line, the radial part of the flank being omitted, or well rounded over if it is desirable to preserve the appearance of the full tooth.

Internal teeth will interfere, even if properly drawn, unless the gear is considerably larger than the pinion running in it. If drawn for the common twelve to rack interchangeable set, there should be at least twelve more teeth in the gear than in the pinion, and if the difference is less, the teeth must be "doctored" or rounded over until they will pass, and there must be a difference of two teeth in any case.

Involute teeth have a decided advantage over epicycloidal teeth for internal gearing, their action being much more direct, with less sliding and friction.

STRENGTH AND HORSE-POWER OF GEARS.

There are about as many different rules for this purpose, and contradictory results, as there are writers upon the subject. I have preferred not to discuss the theory, but to adopt without question the method given by Thomas Box in his Practical Treatise on Mill Gearing, because that engineer has most carefully considered the practical points in view, and because his formulæ agree almost exactly with a great many cases in actual practice.

STRENGTH OF A TOOTH.—For worm gears, crane gears, and slow-moving gears in general, we have to consider only the dead weight that the tooth can lift with safety.

If we allow the iron to be subjected to but one tenth of its breaking strain, we can use the formula:—

$$W = 350\ c\ f,$$

in which W is the dead weight to be lifted, c is the circular pitch, and f the face, both in inches.

For the wooden cogs of mortise wheels, use 120 instead of 350 as a factor in the formula.

When the pinion is large enough to insure that two teeth shall always be in fair contact, the load, as found by this rule, may be doubled.

EXAMPLE.—A cast-iron gear of 3″ circular pitch and 6″ face will lift

$$W = 350 \times 3 \times 6 = 6300 \text{ lbs.}$$

HORSE-POWER OF A GEAR.—For very low speeds we can use the formula,

$$\text{HP for low speed} = .0037\ d\ n\ c\ f,$$

in which d is the pitch diameter, c the circular pitch, and f the face, all in inches, and n is the number of revolutions per minute.

EXAMPLE.—The horse-power of a gear of three feet diameter, three inch pitch, and ten inch face, at eight revolutions per minute, is,

$$HP = .0037 \times 36 \times 8 \times 3 \times 10 = 32.$$

For ordinary or high speeds, where impact has to be considered, it is found that the above formula gives too high results, and we must use the formula,

$$\text{HP at ordinary speeds} = .012\ c^2\ f\ \sqrt{d\,n}.$$

EXAMPLE.—A gear of three feet diameter, three inch pitch and ten inch face, at one hundred revolutions per minute, will carry but

$$HP = .012 \times 9 \times 10 \times \sqrt{100 \times 36} = 65 \text{ horse-power},$$

instead of the 400 horse-power found by the rule for low speeds.

At ordinary or high speeds a wooden cog, on account of its elasticity, will carry as much as or more power than a cast-iron tooth, and we can use .014 instead of .012 in the formula.

When in doubt as to whether a given speed is to be considered high or low, compute the horse-power by both formulæ, and use the smallest result.

For bevel gears the same rules will apply, if we use the pitch diameter and the pitch at the center of the face.

Some rules in use take no account of the face of the gear, but assume that the tooth should be able to bear the whole strain upon one corner.

A tooth that does not bear substantially along its whole face, at several points at least, is a very poor piece of work, and it would be better to straighten the tooth than to force the rule to follow it.

THE EQUIDISTANT SERIES.

The shape of a tooth is not the same on two gears of different sizes, for its curvature continually decreases and the curve flattens as the number of teeth in the gear increases.

When the teeth are formed by a rotary milling tool, we must use a cutter of fixed shape; when formed by planing, a fixed guide is employed; and when drawn by an odontograph, fixed tabular data are used; and obviously, if we require the greatest possible accuracy we must have a different shape of cutter, or guide, or a separate tabular number, for each separate tooth, and at least two hundred in a set to cover the ordinary range of work. As this would be an expensive and clumsy system, it is customary to make one fixed shape do duty for several teeth, being just right for one tooth of a given interval, and approximately so for several teeth either way.

This set of fixed intervals is known as the equidistant series, as it so distributes the errors that the greatest error is the same in all the intervals.

The equidistant series was invented by Willis, but he gives no rule for arranging it, and the example he gives was apparently found by some experimental method.

In the American Machinist for Jan. 8th, 1881, I proposed the location of the dividing points of the series by the formula

$$t = \frac{a\,n}{n - s + \frac{a\,n}{z}}$$

in which a is the first and z the last tooth, usually twelve and infinity, of a series of n intervals. s is the number, in the series, of any particular interval, and t is the last tooth in the interval s.

This formula uniformly distributes, not the differences in form, but what is for all practical purposes the same thing and much more easily handled, the differences in the lengths of the addendum arcs. It is general in its nature, and independent both of the form of the tooth and of its length, which have but a minute effect on the required series. Any method that recognizes these small differences must necessarily require more intricate and difficult trigonometrical work than the slightly increased accuracy will warrant.*

* In his treatise on Kinematics, Prof. C. W. MacCord has treated my formula in such a summary and unjust manner, that in replying to him I do not feel bound by the usual rules of courtesy, but am at liberty to state the facts in plain words, without fear or favor.

He not only refers to my process with an evident attempt at ridicule, but he positively mangles the facts. He is careful to show its defects and to exagerate their importance, while he is equally careful to slight and conceal its real merit. His motive is evident when he next proposes as a substitute a "locus" method which he claims is "the perfect solution of the problem," which will give a series that is "exact to a single tooth," and the value of which he assumes but does not attempt to prove. It is, in fact, an arbitrary approximation, and so wonderfully intricate, clumsy, and inaccurate, that the result, determined by it with great care by its own expert inventor, does not divide the locus curve to a single tooth, or in some parts, within several teeth, or distribute the errors of form any more uniformly than does the method it was intended to displace. A full (and free) discussion of this matter may be found in several letters published in the American Machinist in 1884. The locus method gives a result almost identical (for cases in actual use) with the series found by my formula, and if the slight difference can be proved to be in its favor, as has not been done, it is of imperceptible importance, and no offset whatever to the excessive intricacy of the method.

I did not claim perfection for my formula, or imagine it worth the notice that has been taken of it, and I would not in ordinary cases criticise the work of any other writer, but as I have been used with unusual and unprovoked severity, I find it necessary to publish this note in self defence. Both sides of the question are now accessible to any one who may be interested, and all I ask or expect is that my work shall be treated with ordinary fairness, and allowed whatever merit it really has.

For the ordinary series of eight intervals, to cover from 12 to ∞ the formula becomes

$$t=\frac{96}{8-s}$$

and if we put s successively equal to 1, 2, 3, 4, 5, 6, 7, and 8, we get the series of last teeth

$13\frac{5}{7}$, 16, $19\frac{1}{5}$, 24, 32, 48, 96 and ∞,

the resulting equidistant series being

12 to 13,	25 to 32,
14 to 16,	33 to 48,
17 to 19,	49 to 96,
20 to 24,	97 to a rack.

Similarly, if we apply the formula from $a=24$ to $z=\infty$, for $n=12$, we get the series adopted above for the involute odontograph table, and it requires but a few figures and a simple operation to apply it to any other case.

POSITION OF THE "PERFECT" TOOTH.

The "perfect" tooth, whose shape does duty for the whole interval, can best be placed, not at the center of the interval, but by assuming the interval to be a short series of two intervals, and adopting the intermediate value. The proper fixed shape for the interval from c to d is that of the tooth found by the formula

$$v=\frac{2cd}{c+d}$$

For the interval from 145 to 288 the perfect tooth is the 193rd, instead of the 216th at the center.

MAXIMUM ERROR OF THE SERIES.

The odontograph gives the correct position of the perfect tooth only, and the point of the tooth at either end of the interval is out of position by the very small amount found by the formula

$$\text{error}=\frac{.182}{pn}$$

in which p is the diametral pitch, and n is the number of intervals in the series. The odontograph table I have given for epicycloidal teeth has twelve intervals, and the greatest error in the position of the point of any tooth drawn by the table is

$$\text{error}=\frac{182}{12p}=\frac{.015}{p}\text{ inch.}$$

This becomes .015 inch for one diametral pitch, and .005 inch for one inch circular pitch and in direct proportion for other pitches.

For involute teeth this formula becomes

$$\text{error}=\frac{.156}{pn}$$

and for the given table having twenty-four intervals the greatest error is .006 inch for one diametral pitch, and .002 inch for one inch circular pitch.

It is thus seen that the number of intervals used is sufficient for all practical purposes, particularly if the error is still further reduced by adopting intermediate tabular numbers for intermediate numbers of teeth.

STANDARD FACES FOR GEAR WHEELS.

It is desirable for the sake of uniformity and interchangeability, to have a regular system or law of fixed relation between the size of the teeth and the width of the face of a gear wheel.

Such a law is recognized and in general use in a loose way, is a law of common sense, in fact, for it is almost invariably the custom to adopt a coarse tooth for a wide face, and although the practice is far from uniform, an examination of a great many cases, selected at random, will show that the "base," or product of the face and pitch, will average very near the number ten for cut iron gears.

It is obvious that a fixed law should accommodate itself to actual practice as nearly as possible, and, adopting ten as a base as rigidly as a proper respect for standard pitches, and convenient fractions for the faces will permit, we can construct the following table for cut iron gears.

Face	Pitch	Base
$\frac{1}{2}$	20	10
$\frac{5}{8}$	16	10
$\frac{3}{4}$	12	9
1	10	10
$1\frac{1}{4}$	8	10
$1\frac{3}{4}$	6	$10\frac{1}{2}$
$2\frac{1}{2}$	4	10

For small cut gears, which are usually made of brass, the weaker metal requires a coarser base, and we can use the number six for the standard.

Face	Pitch	Base
$\frac{1}{8}$	48	6
$\frac{3}{16}$	32	6
$\frac{1}{4}$	24	6
$\frac{5}{16}$	20	$6\frac{1}{4}$
$\frac{3}{8}$	16	6
$\frac{7}{16}$	14	$6\frac{1}{8}$
$\frac{1}{2}$	12	6

In the same way for cast gears we can construct a system on the number three as a base, as follows:—

Face	Circular Pitch	Base
$1\frac{1}{2}$	$\frac{1}{2}$	3
2	$\frac{3}{4}$	$2\frac{2}{3}$
3	1	3
4	$1\frac{1}{4}$	$3\frac{1}{5}$
$4\frac{1}{2}$	$1\frac{1}{2}$	3
5	$1\frac{3}{4}$	$2\frac{6}{7}$
6	2	3
7	$2\frac{1}{4}$	$3\frac{1}{9}$
8	$2\frac{1}{2}$	$3\frac{1}{5}$
8	$2\frac{3}{4}$	$2\frac{10}{11}$
9	3	3